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The Immersed Boundary Method for Two-Dimensional Foam with Topological Changes

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Abstract. We extend the immersed boundary (IB) method to simulate the dynamics of a 2D dry foam by including the topological changes of the bubble network. In the article [Y. Kim, M.-C. Lai, and C. S. Peskin, J. Comput. Phys. 229:5194-5207, 2010], we implemented an IB method for the foam problem in the two-dimensional case, and tested it by verifying the von Neumann relation which governs the coarsening of a two-dimensional dry foam. However, the method implemented in that article had an important limitation; we did not allow for the resolution of quadruple or higher order junctions into triple junctions. A total shrinkage of a bubble with more than four edges generates a quadruple or higher order junction. In reality, a higher order junction is unstable and resolves itself into triple junctions. We here extend the methodology previously introduced by allowing topological changes, and we illustrate the significance of such topological changes by comparing the behaviors of foams in which topological changes are allowed to those in which they are not.

AMS subject classifications: 65-04, 65M06, 76D05, 76M20

Key words: Foam, permeability, capillary-driven motion, immersed boundary method, coarsening, topological changes, T1 and T2 processes.

1 Introduction

We use the immersed boundary (IB) method [10] to simulate the dynamics of a 2D dry foam, including the topological changes of the foam structure. Liquid foam is a gas-filled

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space divided into bubbles or cells of which the boundaries are liquid. In a "dry" foam, in which most of the volume is attributed to its gas phase, the bubbles are nearly polyhedral in shape. The thin liquid boundaries move under the influence of surface tension and have permeability which allows the gas to move through the thin liquid boundaries. Thus the capillarity and gas exchange between bubbles together result in the evolution in bubble size and topological structure [12]. This process is called diffusive coarsening. The diffusive flux of gas through a liquid boundary is proportional to the pressure difference between the two bubbles that are separated by that boundary.

In [5], we have applied and extended the immersed boundary (IB) method to simulate the dynamics of a dry foam in the two-dimensional case, and tested it by verifying the von Neumann relation [8,9]: let A be the area of one bubble of a foam, then

$$\frac{dA}{dt} = -M\gamma \int_\Gamma \kappa dl = -2\pi M\gamma \left(1 - \frac{n}{6}\right), \tag{1.1}$$

where M, γ, κ, and n are the permeability coefficient, surface tension, curvature of the bubble boundary Γ, and the number of vertices of the bubble, respectively. The von Neumann relation simply says that the area is constant for 6-sided bubbles, that bubbles with fewer than 6 sides tend to disappear (and in fact reach zero area in finite time), and that bubbles with more than 6 sides tend to grow; hence the "coarsening" of the foam, in which bubbles with large numbers of sides grow at the expense of bubbles with small numbers of sides.

We verified numerically the von Neumann relation in [5] and we showed there the capability of the IB method to simulate the dynamics of a foam with an arbitrarily general shape which experiences the coarsening due to the gas diffusion through the bubble boundaries. An important limitation of the numerical method used in [5], however, was that we did not allow for the resolution of quadruple or higher order junctions into triple junctions. The total disappearance of a bubble with four or more edges generates quadruple or higher order junction at which the exterior angles are no longer equal to $2\pi/6$, as was assumed in the derivation of the von Neumann relation (1.1). In reality, higher order junctions are unstable and resolve themselves into a pair of triple junctions, with the creation of a new edge that arises at zero length and grows in length as the two triple junctions move apart.

We now extend the methodology of [5] and introduce a method to resolve quadruple or higher order junctions into triple junctions. The extension is based on two primary topological changes which may occur in the evolution of a bubble network [3,12]. The first one, which is called a T1 process, is needed when an edge of a bubble gradually shrinks and is about to make a quadruple junction. Since a quadruple junction is unstable, after some transient, this vertex decomposes to form two triple junctions, but in a different configuration. The second one, which is called a T2 process, occurs when a three-edged bubble gradually shrinks and finally disappears. Other topological changes are also possible through the combination of these two processes. For example, a four-sided bubble can disappear by a T1 process followed by a T2 process.