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# Non-Matching Grids for a Flexible Discretization in Computational Acoustics

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**Abstract.** Flexible discretization techniques for the approximative solution of coupled wave propagation problems are investigated. In particular, the advantages of using non-matching grids are presented, when one subregion has to be resolved by a substantially finer grid than the other subregion. We present the non-matching grid technique for the case of a mechanical-acoustic coupled as well as for acoustic-acoustic coupled systems. For the first case, the problem formulation remains essentially the same as for the matching situation, while for the acoustic-acoustic coupling, the formulation is enhanced with Lagrange multipliers within the framework of Mortar Finite Element Methods. The applications will clearly demonstrate the superiority of the Mortar Finite Element Method over the standard Finite Element Method both concerning the flexibility for the mesh generation as well as the computational time.

**AMS subject classifications**: 65L60, 74S05

**Key words**: Nonmatching grids, Mortar FEM, computational acoustics, piezoelectric actuators.

## 1 Introduction

In many engineering applications vibrations are responsible for the generation of acoustic noise. Especially slender or thin-walled structures with a large surface exhibit such a behavior. A modern way of controlling those vibrations is to attach piezoelectric patches to membrane/plate like structures which can measure their deformations and by using adequate power electronics act against the vibrations. These enhanced devices are so-called smart materials.

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It is our goal to simulate such devices by applying the Finite Element Method (FEM). The standard method does not offer enough flexibility to freely place the piezoelectric actuators on the membrane/plate structures. We therefore resort to use the Mortar FEM. In both structures we solve for the partial differential equation (PDE) describing the effects of linear elasticity. In the piezoelectric actuator the electric-mechanical coupling has to be taken into account additionally. The continuity of the normal stresses between the membrane/plate structure and the piezoelectric actuator is maintained by introducing a Lagrange multiplier. Now, the discretizations of both parts need not match on the common surfaces any more and we are therefore allowed to freely place the piezoelectric actuators on the membranes/plates. Therefore, we have to deal with the situation of nonconforming grids appearing at the common interface of two subdomains. Special care has to be taken in order to define and implement the appropriate discrete coupling operators (see, e.g., [2,4,6,11,17]).

In this contribution we extend our research first published in [9] to full multiphysics application including nonmatching mechanical-mechanical and mechanical-acoustic interfaces. Therewith, we apply the method to practically relevant application, e.g., piezoelectric patches attached to mechanical structures for active vibration as well as noise control. In order to simulate the noise radiated from a vibrating structure we once again apply nonmatching grids and extend the computational mesh for the plate by a mesh for acoustic propagation. In this case however, no Lagrange multiplier is required since the coupling takes place between two different physical quantities (mechanical displacement and acoustic pressure).

The rest of this paper is organized in the following way. In Section 2 we introduce the basic equations of linear piezoelectricity, the coupling scheme for mechanics on nonmatching grids, and the coupling between the mechanical field with the acoustic field. In Section 3 we describe the application of our enhanced scheme for the numerical computation of a metal plate with attached piezoelectric patches. A summary and conclusions are given at the end.

## 2 Governing equations and numerical scheme

### 2.1 Equations of linear piezoelectricity

The linearized material law describing the piezoelectric effect is given by [13]

$$\boldsymbol{\sigma} = [\boldsymbol{c}^E]\mathbf{S} - [\boldsymbol{e}]^{\mathrm{t}}\mathbf{E}, \tag{2.1}$$

$$\mathbf{D} = [\boldsymbol{e}]\mathbf{S} + [\varepsilon^S]\mathbf{E}. \tag{2.2}$$

Here $\boldsymbol{\sigma}$ is the tensor of mechanical stresses in Voigt notation, $[\boldsymbol{c}^E]$ the linear stiffness tensor at constant electric field, $\mathbf{S}$ denotes the tensor of mechanical strains (also in Voigt notation), $[\boldsymbol{e}]$ the tensor of piezoelectric coupling coefficients ($\square^{\mathrm{t}}$ denotes the transposed), $\mathbf{E}$ the electric field vector, $\mathbf{D}$ the vector of the electric flux density and finally $[\varepsilon^S]$ the tensor