Commun. Comput. Phys.
doi: 10.4208/cicp.200110.040511a

Vol. **11**, No. 3, pp. 925-950
March 2012

An Immersed Interface Method for the Simulation of Inextensible Interfaces in Viscous Fluids

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Received 20 January 2010; Accepted (in revised version) 4 May 2011

Communicated by Jaw-Yen Yang

Available online 28 October 2011

Abstract. In this paper, an immersed interface method is presented to simulate the dynamics of inextensible interfaces in an incompressible flow. The tension is introduced as an augmented variable to satisfy the constraint of interface inextensibility, and the resulting augmented system is solved by the GMRES method. In this work, the arclength of the interface is locally and globally conserved as the enclosed region undergoes deformation. The forces at the interface are calculated from the configuration of the interface and the computed augmented variable, and then applied to the fluid through the related jump conditions. The governing equations are discretized on a MAC grid via a second-order finite difference scheme which incorporates jump contributions and solved by the conjugate gradient Uzawa-type method. The proposed method is applied to several examples including the deformation of a liquid capsule with inextensible interfaces in a shear flow. Numerical results reveal that both the area enclosed by interface and arclength of interface are conserved well simultaneously. These provide further evidence on the capability of the present method to simulate incompressible flows involving inextensible interfaces.

AMS subject classifications: 65N06, 35R05, 65M12

Key words: Inextensible interface, Stokes flows, singular force, immersed interface method, CG-Uzawa method, front tracking.

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1 Introduction

The membrane of biological cells consisting of lipid bilayers has much attention due to the occurrence in many biological phenomena [28] and used widely as model for the red blood cells [24] and drug-carrying capsules [30]. Most biological membranes can deform but resist area dilation and are often modelled as inextensible interfaces with the position-dependent tension playing the role of surface pressure [41]. To account for the interface inextensibility or incompressibility, the tension is an unknown quantity which is to be computed as part of the solution so as to satisfy the condition of inextensibility. In another word, this is to ensure that the arclength of an arbitrary element of the membrane is conserved during the motion. The viscous flows outside and inside the membrane can also be treated as comprising similar or different incompressible fluids.

Peskin's immersed boundary method (IBM) [22] has been applied widely for simulating such biological flows with moving interfaces. The method was originally developed to study the fluid dynamics of blood flow in a human heart [21], and has further been developed for a wide variety of applications; in particular for the biological problems where complex geometries and immersed elastic interfaces are present, such as the deformation of red blood cells in a shear flow [7], swimming of organisms [9], platelet aggregation [10, 11, 37], and cochlear dynamics. Other applications can be found in [2], biofilm processes [6], wood pulp fiber dynamics [25], and with a more extensive list given in [22]. In the IBM, the force densities are spread to the Cartesian grid points by a discrete representation of the delta function. The fluid equations with the forcing terms are then solved for the pressure and the velocity at the mentioned Cartesian grid points. The resulting velocities are then interpolated back to the control points using the same set of discrete delta functions. Since the immersed boundary method uses the discrete delta function approach, it smears out sharp interfaces and it is of first-order accuracy in space.

In order to capture the jumps in the solution across the interface, the immersed interface method (IIM) incorporates the known jumps into the finite difference scheme near the interface. As such, the IIM avoids smearing out the sharp interfaces and maintains a second-order accuracy. The IIM was originally proposed by LeVeque and Li [15] for solving elliptic equations, and later extended to Stokes flow with elastic boundaries or surface tension [14]. The method was further developed for the Navier-Stokes equations in [12, 13, 17, 19, 39]. The IIM was also used in [4, 18, 26] for solving the two-dimensional streamfunction-vorticity equations on irregular domains. Tan et al. [36] developed an IIM for the Stokes equations on irregular domain. In [35], Tan et al. developed an IIM for the Navier–Stokes equations with discontinuous viscosity across the interface. Xu and Wang [40] extended the IIM approach to the 3D Navier-Stokes equation for simulating the fluid-solid interaction. The interested readers are referred to the recently published book by Li and Ito [16] and the references therein for more applications of the IIM.

In [41], Zhou and Pozrikidis studied the deformation of inextensible interfaces based on the boundary element formulation. A boundary integral method developed in [33] for simulating the dynamics of inextensible vesicles suspended in a viscous fluid in 2D is