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NUMERICAL CALCULATION OF EFFECTIVE PERMEABILITY BY DOUBLE RANDOMIZATION MONTE CARLO METHOD

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Abstract. The paper is devoted to solving the boundary value problems with random parameters. We consider flows in a porous medium with random permeability field or random boundary conditions. The Monte Carlo method with double randomization is suggested to compute the statistical properties of the flow. The paper focuses on the calculating of media's effective permeability. The method is compared with a standard Monte Carlo approach. Numerical tests show that double randomization gives high accuracy and can improve computational efficiency.

Key words. Stochastic models, effective permeability, Monte Carlo method, double randomization

1. Introduction

Many problems in natural science, industry and finance are naturally described by stochastic models. For instance, such models are used in simulation of turbulent transport [5] or evaluation of the elastic properties of composite materials [14, 25]. In [8] the transport in a random magnetic field is studied. The stochastic modeling of bacterial population dynamics is considered in [16, 28].

The main purpose of this paper is to develop an effective numerical method for solving the problems described by partial differential equations with random parameters. In such equations coefficients, right side or boundary conditions can be considered as random functions. Certainly, in statistical approach we are able to evaluate only some averaged flow characteristics.

One of the most important applications of this method is a simulation of the flow in porous media. In many articles permeability is approximated by random field [3, 10, 11]. In this paper we address the calculation of effective permeability used for the solution of filtration problems [21, 22]. In particular, the developed method can be used to study the influence of deformation bands distribution on fluid flow in fault damage zone [13]. Several approaches are developed for solving such problems. The small perturbation expansion method [2, 10, 27] is computationally efficient but it is restricted by values of permeability fluctuations. The applicability of the spectral model derived under the assumption of small hydraulic conductivity fluctuations is studied in [12]. In [30] the mean and covariance of hydraulic head for saturated flow in randomly heterogeneous porous media is calculated by using the Karhunen-Loeve decomposition, polynomial expansion, and perturbation method. The uncertainty analysis of flow in random porous media is explored in [17] by probabilistic collocation method. These techniques have relatively low computational cost but it also requires the Karhunen-Loeve decomposition for covariance function of permeability random field. A vast review devoted to stochastic computations presented in [29].

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The most general and popular approach is as follows:

1. the ensemble of realizations of the random parameter is sampled;
2. the deterministic equation for each realization of parameters is solved numerically.

Then the desired averaged flow characteristics are evaluated by using ensemble averaging. Further we will call this procedure as a "standard approach". Unfortunately, this method may be very time consuming. In this work we use the "double randomization" method [19, 24] to overcome this difficulty. As a standard approach, this technique also has no restrictions on the permeability distribution. We consider a standard Monte Carlo approach and the Monte Carlo method, which uses the double randomization for calculating the effective permeability of coarse grid block. The method's efficiency is compared by using two different models of permeability distributions.

2. Formulation of the problem

We consider a steady flow through a saturated porous medium. For a stationary 2D flow, we solve the following Darcy law and the continuity equation:

$$\mathbf{q} = -\frac{1}{\mu} K \nabla p, \tag{1}$$

$$\nabla \cdot \mathbf{q} = 0 \tag{2}$$

where $\mathbf{q}$ is the Darcy velocity, p is the pressure, μ is the dynamic viscosity (constant in all the simulations, $\mu = 1 Pa \cdot s$) and K is the permeability. Here and below in the paper we use bold font for vector variables and matrixes.

Due to the strongly irregular structure of the media, we assume that the permeability field is a random field. Then any flow characteristic ξ (flow rate, velocity, effective permeability etc.) also becomes random function defined as solution of (1), (2). Certainly, this approach allows us to compute averaged flow characteristics only. Having the ensemble of the random fields realizations sampled according to the correspondent distribution, we can calculate the value of the flow characteristic ξ_i for each realization, as well as the effective properties by using the following statistical averaging:

$$E\xi = \langle \xi \rangle \approx \frac{1}{N} \sum_{i=1}^{N} \xi_i \tag{3}$$

where N is the number of realizations. Here $\langle\rangle$ means the ensemble averaging.

We solve equations (1), (2) in the domain $\Omega = \{0 \leq x \leq L_x, 0 \leq y \leq L_y\}$. For simplicity, the effective permeability will be calculated by using the upscaling procedure in one direction described in [6, 7]. On two opposite boundaries, the pressures are fixed to constant values $p(0, y)$ and $p(L_x, y)$, whereas no flow boundary conditions apply to the other borders. The flow calculated numerically allows us to estimate the effective permeability K_{eff} of a coarse upscaled block Ω from equation [22]

$$\langle \mathbf{q} \rangle = -\frac{1}{\mu} \langle \nabla p \rangle. \tag{4}$$