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DEVELOPMENT OF A *hp*-LIKE DISCONTINUOUS GALERKIN TIME-DOMAIN METHOD ON NON-CONFORMING SIMPLICIAL MESHES FOR ELECTROMAGNETIC WAVE PROPAGATION

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Abstract. This work is concerned with the design of a *hp*-like discontinuous Galerkin (DG) method for solving the two-dimensional time-domain Maxwell equations on non-conforming locally refined triangular meshes. The proposed DG method allows non-conforming meshes with arbitrary-level hanging nodes. This method combines a centered approximation for the evaluation of fluxes at the interface between neighboring elements of the mesh, with a leap-frog time integration scheme. It is an extension of the DG formulation recently studied in [13]. Several numerical results are presented to illustrate the efficiency and the accuracy of the method, but also to discuss its limitations, through a set of 2D propagation problems in homogeneous and heterogeneous media.

Key Words. Maxwell's equations, discontinuous Galerkin method, *hp*-like method, non-conforming triangular mesh, computational electromagnetism.

1. Introduction

The difficulties linked to the numerical solution of the time-domain Maxwell equations are to find their roots in the characteristics of the underlying wave propagation problems, *i.e.* the geometry of the diffracting objects, the physical features of the propagation medium (heterogeneity, physical dispersion and dissipation) and the type of sources (wires, etc.). Applications with such characteristics can be found throughout the applied sciences and engineering, *e.g.* the design and optimization of antennas [5] and radars [21], the design of emerging technologies such as high speed electronics and integrated optics, and a variety of military and civilian applications [22]-[20]. Other challenging applications are addressing societal questions such as the potential adverse effects of electromagnetic waves emitted by mobile phones [24]. Such problems require high fidelity approximate solutions with a rigorous control of the numerical errors. Even for linear problems such conditions force one to look beyond standard computational techniques and seek new numerical frameworks enabling the accurate, efficient, and robust modeling of wave phenomena over long simulation times in settings of realistic geometrical complexity.

The finite difference time-domain (FDTD) method, first introduced by Yee in 1966 [33] and later developed by Taflove and others [28], has been used for a broad range of applications in computational electromagnetics. In spite of its flexibility and second-order accuracy in a homogeneous medium, the Yee scheme suffers from serious accuracy degradation when used to model curved objects or when treating material interfaces. Indeed, the so-called staircasing approximation may

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lead to local zeroth-order and at most first-order accuracy; it may also produce locally non-convergent results [11]. Furthermore, for Maxwell's equations with discontinuous coefficients, the Yee scheme might not be able to capture the possible discontinuity of the solution across the interfaces [11]. A number of finite difference methods have been proposed in the past for the treatment of curved interfaces. The usual and straightforward approach is to introduce local modification into the Yee scheme but still keep the staggered grid [12]-[31], or to use local mesh refinements [9]. Recently, some studies have been concerned with high-order embedded FDTD schemes in the presence of material interfaces [16], including the staggered fourth-order accurate methods by Yefet *et al.* [29]-[34], and the fourth-order orthogonal curvilinear staggered grid methods by Xie *et al.* [32]. Also, high-order FDTD methods via hierarchical implicit derivative matching are presented in [36]. Most of these methods, however, have not really penetrated into main stream user community, partly due to their complicated nature and partly because these new methods themselves often introduce other complications.

Finite element methods can handle unstructured meshes and complex geometries but the development of such methods for solving Maxwell's equations, especially those with high-order accuracy, has been relatively slow. A primary reason is the appearance of spurious, non-physical solutions when a straightforward nodal continuous Galerkin finite element scheme is used to approximate the Maxwell curl-curl equations. Bossavit made the fundamental observation that the use of special curl-conforming elements [19] would overcome the problem of spurious modes by mimicking properties of vector algebra [3]. Although very successful, such formulations are not entirely void of problems: the algebraic problems are larger than for nodal elements and the conformity requirements of the continuous Galerkin formulation makes adaptivity a complex task.

In an attempt to offer an alternative to the classical finite element formulation based on edge elements, we consider here discontinuous Galerkin formulations [6] based on high-order nodal interpolation for solving the time-domain Maxwell equations in first-order form. Discontinuous Galerkin time-domain (DGTD) methods based on discontinuous finite element spaces, easily handle elements of various types and shapes, irregular non-conforming meshes [13], and even locally varying polynomial degree, and hence offer great flexibility in the mesh design. They also lead to (block-) diagonal mass matrices and therefore yield fully explicit, inherently parallel methods when coupled with explicit time stepping [2]. In fact, for constant material coefficients, the mass matrix is diagonal for a judicious choice of (locally orthogonal) shape functions [23]. Moreover, continuity is weakly enforced across mesh interfaces by adding suitable bilinear forms (so-called numerical fluxes) to the standard variational formulations. Whereas high-order discontinuous Galerkin time-domain methods have been developed on conforming hexahedral [8] and tetrahedral [14]-[15] meshes, the design of non-conforming discontinuous Galerkin time-domain methods is still in its infancy. In practice, the non-conformity can result from a local refinement of the mesh (*i.e.* h-refinement), of the interpolation degree (*i.e.* p-enrichment) or of both of them (*i.e.* hp-refinement).

This work is a continuation of [13] where a h-refinement DGTD-$\mathbb{P}_p$ method was introduced for solving the two-dimensional time-domain Maxwell equations on non-conforming triangular meshes. The remaining of this paper is organized as follows. In section 2 we recall the basic features of the discontinuous Galerkin time-domain formulation for solving the two-dimensional Maxwell equations in first-order form, based on totally centered numerical fluxes and a leap-frog time-integration scheme.