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Amplitude Compensation for One-Way Wave Propagators in Inhomogeneous Media and its Application to Seismic Imaging

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Abstract. The WKBJ solution for the one-way wave equations in media with smoothly varying velocity variation with depth, $c(z)$, is reformulated from the principle of energy flux conservation for acoustic media. The formulation is then extended to general heterogeneous media with local angle domain methods by introducing the concepts of Transparent Boundary Condition (TBC) and Transparent Propagator (TP). The influence of the WKBJ correction on image amplitudes in seismic imaging, such as depth migration in exploration seismology, is investigated in both smoothly varying $c(z)$ and general heterogeneous media. We also compare the effect of the propagator amplitude compensation with the effect of the acquisition aperture correction on the image amplitude. Numerical results in a smoothly varying $c(z)$ medium demonstrate that the WKBJ correction significantly improves the one-way wave propagator amplitudes, which, after compensation, agree very well with those from the full wave equation method. Images for a point scatterer in a smoothly varying $c(z)$ medium show that the WKBJ correction has some improvement on the image amplitude, though it is not very significant. The results in a general heterogeneous medium (2D SEG/EAGE salt model) show similar phenomena. When the acquisition aperture correction is applied, the image improves significantly in both the smoothly varying $c(z)$ medium and the 2D SEG/EAGE salt model. The comparisons indicate that although the WKBJ compensation for propagator amplitude may be important for forward modeling (especially for wide-angle waves), its effect on the image amplitude in seismic imaging is much less noticeable compared with the acquisition aperture correction for migration with limited acquisition aperture in general heterogeneous media.

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Key words: Propagator amplitude, image amplitude, WKBJ, beamlet, local angle domain, seismic imaging.

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1 Introduction

Full wave equation based modeling methods (e.g., finite-difference, finite-element) can model all wave phenomena and provide accurate amplitudes of the wavefield, but are computationally expensive, especially for the 3D case. Asymptotic theory based methods, such as the ray method, have been widely used in industry due to their efficiency, straightforward implementation and flexibility in target-oriented imaging. The high-frequency approximation limits their application in complicated regions. One-way wave equation (parabolic wave equation) based propagators provide powerful and fast tools for forward modeling and migration. They are much faster than full wave methods and can handle complex velocity models better than ray-based methods. The original one-way wave equations, introduced to exploration geophysics by Claerbout [1, 2], do not provide accurate amplitudes even at the level of leading order asymptotic WKBJ or ray-theoretical amplitudes [3].

The conventional one-way wave equation based migration can offer reflector maps consistent with real subsurface structures, but may provide unreliable reflection/scattering strength (or image amplitude) of the reflectors/scattering objects. True-reflection (or true-amplitude) imaging tries to give not only correct location but also correct image amplitude of the reflectors. This can bridge the gap between the conventional imaging and direct inversion of the medium parameters. There are many factors which may influence the image amplitude, including propagator errors (e.g., focusing and defocusing by heterogeneity, geometrical spreading, path absorption and path scattering loss, numerical dispersion and numerical anisotropy), acquisition aperture effects, imaging conditions, etc.

The amplitude errors in the conventional one-way wave propagators have been studied by different approaches for a long time [3–10]. The WKBJ correction can be introduced into the one-way wave equations to compensate the amplitude of the propagators. Traditionally, the WKBJ solution is derived by asymptotic approximation in smoothly varying $c(z)$ media [11–14]. It has also been obtained by approximately factorizing the full-wave operator into one-way wave operators in heterogeneous media [4]. With an extra amplitude term introduced to the conventional one-way wave equations, the first-order transport equation of the one-way wave equation will be the same as that from the full-wave equation in the sense of high-frequency approximation. In this sense, they are called ”true-amplitude” one-way wave equations [3].

Understanding the reciprocity properties of (forward and inverse) one-way propagators is relevant for the design of true-amplitude migration schemes [7]. The usual one-way propagators are pressure-normalized: the sum of the downgoing and upgoing waves equals the acoustic pressure of the total wavefield, and they do not obey reciprocity [7]. The so called flux-normalized [5–7,15–17] one-way propagators, in which the transmission coefficients at an interface for downgoing and upgoing waves are identical, on the other hand, do obey reciprocity [7]. For horizontally layered media there is a vast amount of literature that makes use of the flux-normalized decomposition [15–17]. It can