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# Three-Dimensional Lattice Kinetic Scheme and its Application to Simulate Incompressible Viscous Thermal Flows

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**Abstract.** In this paper, the three-dimensional lattice kinetic scheme is presented to simulate incompressible viscous thermal flows. As compared with the standard LBM, the present scheme has the following good features. It can save the computer memory since there is no need to store the density distributions. Like the conventional NS solvers, the implementation of boundary conditions is straightforward since the dependent variables are the macroscopic flow parameters. The easy implementation of boundary conditions is a good property for solving three-dimensional flow problems. The present scheme is validated by simulating the three-dimensional natural convection in an air-filled cubical enclosure, which is heated differentially at two vertical side walls. The obtained numerical results compare very well with available data in the literature.

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**Key words**: Lattice kinetic scheme, lattice Boltzmann method, three-dimensional, incompressible flow, natural convection, thermal flow.

## 1 Introduction

The lattice Boltzmann method (LBM) is an alternative numerical scheme for simulating viscous flows [1, 2]. It has been widely used in many kinds of complex flows such as the turbulent flow, multiphase flow and micro-flow [3]. It has the following good features: the linear convection operator in the phase space, the pressure calculated using an equation of state and the use of a minimal set of velocities in the phase space. Furthermore, it

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has the intrinsic feature of parallelism. The only variables for LBM are the density distributions. The mapping from the density distributions to the fluid variables is a straightforward summation. In contrast, the inverse mapping from the fluid variables to the density distributions is somewhat tricky to make, especially for the three-dimensional problem. However, on the boundaries, the macroscopic fluid variables, instead of the density distributions, are usually given as the boundary conditions. The inverse mapping has to be used on the boundaries. Different ways to make the inverse mapping form different methods to implement boundary conditions. The bounce-back rule for the boundary condition is the simplest way to impose on the solid wall. Particles that meet at a wall point are simply bounced back with a reverse direction. This rule leads to a non-slip boundary located at somewhere between the wall nodes and the adjacent fluid nodes [4]. It is found to be only the first order in the numerical accuracy at the boundaries [5, 6] and its serious shortcomings have been pointed out by Noble et al [7]. More sophisticated boundary conditions, which model the non-slip boundary exactly at the wall nodes, have been proposed by several authors [8, 9]. Among them, the consistent hydrodynamic boundary condition has been widely used in recent years. It calculates the unknown density distributions from the velocity boundary conditions and the density distributions of neighboring fluid nodes near the boundary. This requires that the unknown density distributions should not exceed the available number of equations for the density and momentum. For two dimensions, the number of available equations is three and it is four for three dimensions. However, for the general three-dimensional problems, the unknown density distributions usually exceed four, especially for the corner points. The supplementary rules have to be introduced. Chen et al. [10] proposed a new boundary condition using a second-order extrapolation scheme to obtain the unknown density distributions on the boundary. Bouzidi et al. [11] proposed a new scheme for wall boundary conditions. It uses the bounce-back rule and interpolation. Ginzburg and d'Humiéres [12] presented a general framework for several previously introduced boundary conditions, such as the bounce-back rule and the linear and quadratic interpolations, and designed boundary conditions for general flows which are third-order kinetic accurate. Starting from the well developed theory of boundary conditions for the continuous Boltzmann equation, Ansumali and Karlin [13] derived the boundary condition for the discrete set of velocities. Using this boundary condition, the Knudsen layer in the Kramers' problem is reproduced correctly for small Knudsen numbers.

As an alternative approach, the two-dimensional lattice kinetic scheme was proposed by Inamuro [14]. Similar idea was also proposed by Martys [15]. It is based on the idea that if the dimensionless relaxation time in the LBM with BGK model is set to unity, the macroscopic variables can be calculated without using density distributions and the scheme becomes very similar to the kinetic approach. By merging the LBM with kinetic scheme, the implementation of boundary conditions is very easy and straightforward since on the boundaries, only the macroscopic variables are needed as for the conventional NS solvers. This feature is very distinguished as compared with the conventional LBM when the flow problems with complex geometry are solved. In addition, it can