## Acoustic phonon transmission and thermal conductance in a three-stub quantum waveguide

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**Abstract.** Using the scattering matrix method, we investigate the acoustic phonon transmission and thermal conductance in a quantum waveguide with three stubs at low temperature. It is found that transmission coefficient shows periodic feature and the number of cutoff frequency bands increases with increasing of height h; and the thermal conductance exhibits oscillatory decaying behavior with increasing of the widths between any two stubs; In addition, thermal conductance is sensitive to the width and height of stubs; The results show that changing the geometric parameters of the stubs could provide an efficient way to control the thermal conductance of the proposed micro-structures artificially.

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**Key words**: T-shaped quantum waveguide, scattering matrix method, acoustic transmission coefficient, thermal conductance

## 1 Introduction

Acoustic phonon heat transmission in a quantum waveguide structure is a very important subject and has attracted increasing attention in recent years [1–10]. The characters of phonon transmission and thermal conductance are mainly researched objects because phonon transmission and thermal conductance are two vital parameters in semiconductor nanostructure which plays a key role in controlling the performance and stability of phonon device. Many theoretical and experimental investigations of phonon transport in various kinds of nanostructure such as thin films [2], quantum wells [3], superlattices [4], nanowires [5–7], onedimensional glass [8], and nanotubes [9, 10] have been reported and made tremendous progress in the last decade. Using the Landauer formulation of transport theory, several groups [11–14] have derived the expressions of phonon transmission and thermal conductance for ballistic phonon transport at a low enough temperature in an ideal elastic beam

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and found that the thermal conductance which quantized in a universal value  $\pi^2 \kappa_B T/3h$ , analogous to the well-known  $2e^2/h$  electronic conductance quantum. And they also found that thermal conductance is controlled by the first several modes at low temperature. These predictions have been verified experimentally by Schwab et al. [15]. Li and his copartner researched phonon transmission and thermal conductance in a quantum waveguide structure by using the scattering matrix method recently, then they revealed a series of interesting characters such as acoustic phonon mode splitting behavior [16] and the nonintegral quantized thermal conductance of an asymmetric y-branch three terminal junction [17] at very low temperatures through a T-shaped quantum waveguide. Scattering matrix method is an effective method for accounting of phonon or electron transmission and thermal conductance in nanostructure. Using this method, in this paper, we mainly investigate the characters of phonon transmission and thermal conductance in a three-stub quantum waveguide structure and obtain some novel and interesting physical characteristics which are different from other shaped discontinuity quantum waveguide nanostructure. For example, transmission coefficient shows periodic feature with the increasing of heights h at the different reduced frequencies and thermal conductance exhibits oscillatory decaying behavior with the width between any two stubs.

## 2 Model and formalism

We consider the geometric structure as shown in Fig. 1, which is divided into seven regions (*I-V11*). We assume that the temperature in region  $\xi$  is  $T_{\xi}$  and the temperature difference  $\delta T$  is very small, so the mean temperature (*T*) can be adopted as the temperature of every region in the following calculations. As we known if the thickness of structure is very thin, then we can ignore mode mixing effects at boundaries and interfaces. In really three-dimensional case, then the thickness should be small with respect to the other dimensions and also to the wavelength of the elastic waves. There is no mixing of the *Z* mode, so we only need calculate a two-dimensional case.

We employ the expression of thermal conductance K as [18, 19]

$$K = \frac{\hbar^2}{k_b T^2} \sum_m \frac{1}{2\pi} \int_{\omega_m}^{\infty} \tau_m(\omega) \frac{\omega^2 e^{\beta h\omega}}{(e^{\beta h\omega})^2} d\omega, \tag{1}$$

where  $\tau_m(\omega)$  is the energy transmission coefficient form mode *m* of region *I* at frequency  $\omega$  across all the interfaces into the modes of region *VII*,  $\omega_m$  is the cutoff frequency of the *m*-th mode,  $\beta = 1/k_BT$ ,  $k_B$  is the Boltzman constant, *T* is the temperature and *h* is the Plank's constant. The effect of scattering is introduced through the transmission coefficient, so the central issue in predicting the thermal conductance is then to calculate the transmission coefficient. When waveguide structure is continuous,  $\tau_m(\omega) = 1$ , but if the structure is discontinuity,  $\tau_m(\omega) \leq 1$ , owing to scattering effect in the stubs.

In this paper, we employ the elastic model to calculate the transmission coefficient of acoustic phonon. To our knowledge, there are three different modes in quantum waveguide