INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 5, Number 3, Pages 407–440

COMPUTATIONAL ASPECTS OF LOW FREQUENCY ELECTRICAL AND ELECTROMAGNETIC TOMOGRAPHY: A REVIEW STUDY

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This paper is dedicated to my beautiful wife Leyla

Abstract. This paper studies various mathematical methods for image reconstruction in electrical impedance and magnetic induction tomography. Linear, nonlinear and semilinear methods for the inverse problems are studied. Depending on the application, one of these methods can be selected as the image reconstruction algorithm. Linear methods are suitable for low contrast imaging, and nonlinear methods are used when more accurate imaging results are required. A semilinear method can be used to preserve some properties of the nonlinear inverse solver and at the same time can have some advantages in computational time. Methods design specifically for jump in material distribution as well as dynamical imaging have been reviewed.

Key Words. Electrical impedance tomography, electrical capacitance tomography, magnetic induction tomography, magnetostatic permeability tomography, inverse problems.

1. Introduction

Imaging is the science of building 2D or 3D reconstructions from exterior measurements. The applications include medical and industrial tomography, geomagnetic prospecting, ground penetrating radar, industrial non-invasive testing and many others. Computational imaging algorithms involve the solution of large-scale inverse problems, in the form of constrained or unconstrained optimization problems. Due to the similarities in the underlying mathematical formulations of the problems, one can design algorithms which can be combined to solve large classes of applications problems.

Many imaging problems belong to the general class of the inverse problem, whose solutions are extremely sensitive to data errors (and rounding errors in the computations). An approximated solution to these problems can be computed by incorporating a priori information about the desired solution into the reconstruction model. This information can be defined explicitly, e.g., by requiring the solution to satisfy given constraints or to lie in a given subspace, or the information can be implicit, e.g., by requiring that the solution satisfies certain smoothness conditions. The algorithms that incorporate these requirements into the solution process are called *regularization algorithms*, and they usually take the form of a linear or nonlinear optimization algorithm that involves a combination of a "goodness-of-fit"

Received by the editors Jan 15, 2007 and, in revised form, September 25.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.

(such as the residual norm) and a "quality measure" (such as a semi-norm) of the solution.

Low frequency electromagnetic tomography techniques (less than 20 MHz) are used to non-invasively create cross sectional images of the objects with contrasts in one or more of the passive electromagnetic properties (PEP) including conductivity, permittivity and permeability. Magnetic induction tomography (MIT) [26], [37], [46], [55], [48] is a relatively new member of the electromagnetic imaging family, which works based the eddy current in conductive objects. Image reconstruction of MIT and the three other members of this family, including magnetostatic permeability tomography (MPT) [61], [31], electrical impedance tomography (EIT) [16], [7] and electrical capacitance tomography (ECT) [82] have been studied in this paper.

EIT is the oldest member and was introduced in a medical context by Barber and Brown [6], [30]. ECT has been used for industrial process tomography applications mainly for materials with low permittivity and negligible conductivity. This review will study MIT in conductivity imaging mode and MPT for permeability imaging. EIT considered here works in electrical conductivity mode, so it is referred to as electrical resistance tomography (ERT).

In MIT and MPT a magnetic field from an excitation coil is applied to the object. MIT is based on concept of the eddy current that originates with Michael Faraday's discovery of electromagnetic induction in 1831. In MIT, a time varying magnetic field is induced in the sample material using a magnetic coil with alternating current. This magnetic field causes an eddy current to be generated in conducting materials. These currents, in turn, produce small magnetic fields around the conducting materials. The smaller magnetic fields generally oppose the original field, which changes the transimpedance between excitation and sensing coils. Thus, by measuring the changes in transimpedance between magnetic coils as it traverses the sample, we can identify different characteristics of the sample. In MPT there is no eddy current and changes in magnetostatic fields due to the presence of a permeable object can be detected by sensing coils. The mutual inductances between excitation and sensing coil is the measurement data in MIT. In ERT electrical current is applied to the conductive body via excitation electrodes and resulting electric voltages are measured in peripheral electrodes. In ECT electric potential is applied to the excitation electrodes and capacitances are measured between pairs of electrodes. ERT requires direct contact between the imaging area and the electrodes, but MIT and MPT are fully contactless, and ECT can be used without direct contacts.

All these modalities are inherently complex. They need energization of target region, sensors, electronics, data acquisition and data processing. Induced voltages in MPT and MIT, measured voltages in ERT electrodes and measured capacitances between ECT electrodes are the data for the image reconstruction.

Image reconstruction in EIT is more advanced than for ECT and MIT. For instance, nonlinear image reconstruction methods, including most commonly used regularized Gauss-Newton, are now widely adapted for EIT imaging but not for ECT or MIT. The area of image reconstruction in ECT and MIT is still very underdeveloped. In the past few years many interesting works have been done in the area of sensor design [49], electronic design [79], [37] and basic understanding of the sensitivity maps in MIT [54]. Various types of linear reconstruction methods were used for the image reconstruction of ECT and MIT [83], [38], [8]. In ECT the main focus was to generate images by fast methods, so the computational time