J. At. Mol. Sci.
doi: 10.4208/jams.052313.080313a

Vol. **5** , No. 1 , pp. 33-43
February 2014

Relativistic thermodynamic properties of interacting Fermi gas in a strong magnetic field

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Received 23 May 2013; Accepted (in revised version) 3 August 2013
Published Online 18 November 2013

Abstract. Based on the Mean Field Theory, the relativistic thermodynamic potential function (RTPF) of interacting Fermi gases in a strong magnetic field is derived. On this basis, by taking the interacting term from the RTPF and using the thermodynamic relations, analytical expressions of the energy, the heat capacity and the chemical potential are calculated at low temperatures. And combined with numerical simulation analysis, the effect of interaction on the thermodynamic properties of the system is analyzed and the regulation mechanisms on the interaction impacts of both the magnetic field and the relativistic effect are discussed.

PACS: 05.30.-d, 71.10.Ca, 51.30.+i

Key words: interaction strong magnetic field, relativity effect, Fermi gas

1 Introduction

Since 1995, because of ultracold Fermionic Gas and Bose Gas being imprisoned in laboratory experiments, people are passionate in the ultracold quantum gas theory. Facts indicate, statistical properties of quantum gases are affected by the external potential, particle mass, interaction, scale of particle system, dimensional effect, relativity effect and other influencing factors. Currently, many articles have researched thermodynamic properties of quantum gases. For example, literatures [1-8] have showed important researching results about ultracold Fermi gases; Gupta et al. utilizing the method of quantum statistics and the thermodynamic theory, have discussed the magnetic susceptibility and system's statistic characters of Fermi gases within a strong magnetic field and low temperatures [9, 10]; Wang et al. have studied non-extensive relativistic system's thermodynamic properties and its stability [11,12]; Men et al. have studied the relativistic effects of the thermodynamic properties of Fermi gas in a strong magnetic field and the

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influential mechanisms of temperatures and the magnetic field on the relativistic effects of the system [13]; Schunck gave the experimental testify to the super-fluidity of ultracold Fermi gas, and has reported that atomic gas 6Li can introduce an everlasting and frictionless eddy motion during the process of forming Bose Einstein Condensation (BEC) [14]; Besides, Fan et al. have discussed the relativity effect of Fermi system in the case of strong magnetic field, and also analyzed its influence mechanism thoroughly [15]. However, systems researched are non-interaction. Namely, they neglected interactions between fermions, however, those interactions affect system properties significantly. This paper has took interactions in Fermi system into consideration making calculation closer to fact, explained the RTPF of the system in a strong magnetic field. Based on the RTPF, and using thermodynamic relations, analytical expressions of thermodynamic quality caused by interaction at low temperatures are computed; according to results of numerical simulation, the influence of weak interaction on the system's thermodynamic properties is analyzed, and the regulation mechanisms on the interaction's impacts of both magnetic field and relativity effect are also discussed.

2 The relativistic thermodynamic potential function of interacting Fermi gas in a strong magnetic field

Suppose that the research system is consisted of particles with spin quantum number equating to 1/2, rest mass m, and this system is within an uniform magnetic field with an intensity of B (only has z-direction). In view of the Dirac energy operator and the second order approximation of non-relativity effect, the energy operator of single particle is given

$$H=\frac{1}{2m}[\sigma(p-\frac{e}{c}A)]^2-\frac{p^4}{8m^3c^2}+n\alpha, \tag{1}$$

where, the first term is Pauli energy operator, the second relativistic modification of kinetic energy, and the last one coming from the Mean Field Theory represents interacting energy. In addition, n is particle-number density and $\alpha=4\pi a\hbar^2/m$ is the parameter of interactions. Here a is the s-wave scattering length. Defining the potential function $A_x=-By$, $A_y=A_z=0$, and according to Men et al. [16], when considering inter-particle interactions, the total energy of single fermion in a strong magnetic field can be written as

$$\varepsilon_p=\frac{p^2}{2m}+2n'\sigma B-\frac{p^4}{8m^3c^2}+n\alpha, \tag{2}$$

where, $\sigma=\frac{\hbar e}{2m}$ is Bohr magneton, and $n'=0,1,2,3,\cdots$ is the quantum number. Popularizing the literature [17], the system's thermodynamic potential function is given

$$\Omega=2\sigma B\left\{\frac{1}{2}f(\mu+\frac{p^4}{8m^3c^2}-n\alpha)+\sum_{n'=1}^{\infty}f(\mu+\frac{p^4}{8m^3c^2}-2n'\sigma B-n\alpha)\right\} \tag{3}$$