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A Front-Tracking Method for Motion by Mean Curvature with Surfactant

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Abstract. In this paper, we present a finite difference method to track a network of curves whose motion is determined by mean curvature. To study the effect of inhomogeneous surface tension on the evolution of the network of curves, we include surfactant which can diffuse along the curves. The governing equations consist of one parabolic equation for the curve motion coupled with a convection-diffusion equation for the surfactant concentration along each curve. Our numerical method is based on a direct discretization of the governing equations which conserves the total surfactant mass in the curve network. Numerical experiments are carried out to examine the effects of inhomogeneous surface tension on the motion of the network, including the von Neumann law for cell growth in two space dimensions.

AMS subject classifications: 65M06

Key words: Front-tracking method, motion by mean curvature, triple-junction, surface tension, surfactant.

1 Problem description

Interface phenomena have been studied extensively not only due to their importance in applications but also for the computational challenges they impose. Motion by mean curvature has been used as a model for various physical problems including multiphase flows and growth of grain boundary in poly-crystals [3]. Level-set method is a popular choice for these type of problems [9]. In [1], a direct finite difference

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method was used to study the evolution of a network of curves due to its simplicity and efficiency.

In this paper, we generalize the grain growth problem discussed in [1] by including the effect of an inhomogeneous surface tension. For practical problems, it is difficult to maintain constant surface tension as insoluble surface active agents (surfactant) are common and their presence could significantly affect the value of the surface tension, therefore the dynamics of interface motion [2]. To account for the effect of the surface tension on the interfacial dynamics of a complex network of interfaces, we consider a network of curves in a two dimensional setting and assume that there is a surfactant distributed along the curve and the surface tension varies according to the surfactant concentration. As in [1], we consider the situation with triple-junctions, i.e., three phase boundaries, described by parametric curves $\mathbf{X}^i(s,t), s\in[0,1]$ for $i=1,2,3$ as shown in Fig. 1. Throughout this paper, we define $\boldsymbol{\tau}^i(s,t)=\mathbf{X}^i_s/|\mathbf{X}^i_s|$ as the unit tangent vector of the curve i. The motion of equations are defined as

$$\mathbf{X}^i_t = \sigma^i \frac{\mathbf{X}^i_{ss}}{|\mathbf{X}^i_s|^2}, \tag{1.1}$$

where $\sigma^i(s,t)$ is the surface tension along the curve $\mathbf{X}^i$ and is determined by the surfactant concentration $\Gamma^i(s,t)$. By taking inner product with normal vector $\boldsymbol{n}^i=\mathbf{X}^{i\perp}_s/|\mathbf{X}^i_s|$, Eq. (1.1) becomes

$$\mathbf{X}^i_t \cdot \boldsymbol{n}^i = \sigma^i \frac{\mathbf{X}^i_{ss}}{|\mathbf{X}^i_s|^2} \cdot \frac{\mathbf{X}^{i\perp}_s}{|\mathbf{X}^i_s|} = \sigma^i \kappa^i, \tag{1.2}$$

where κ^i is the local mean curvature of the curve X^i. Thus the normal velocity of curve motion is proportional to the local mean curvature and the surface tension coefficient.

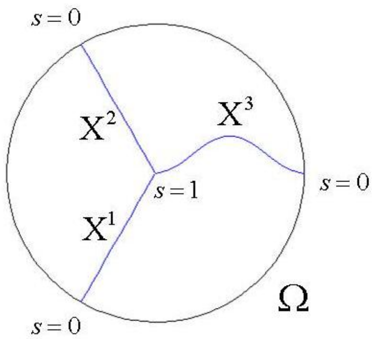

Fig. 1: A three-curve network.

The presence of surfactant reduces the surface tension, and in this paper we use the simplified nonlinear Langmuir equation of state [10]

$$\sigma^i = \sigma_c(1 + \ln(1 - \beta^i \Gamma^i)). \tag{1.3}$$