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A FEASIBLE SEMISMOOTH GAUSS-NEWTON METHOD FOR SOLVING A CLASS OF SLCPS*

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Abstract

In this paper, we consider a class of the stochastic linear complementarity problems (SLCPs) with finitely many elements. A feasible semismooth damped Gauss-Newton algorithm for the SLCP is proposed. The global and locally quadratic convergence of the proposed algorithm are obtained under suitable conditions. Some numerical results are reported in this paper, which confirm the good theoretical properties of the proposed algorithm.

Mathematics subject classification: 90C33, 65K10.
Key words: Stochastic linear complementarity problems, Gauss-Newton algorithm, Convergence analysis, Numerical results.

1. Introduction

Assume that $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space with $\Omega \subseteq \Re^n$, where the probability distribution $\mathcal{P}$ is known. The stochastic linear complementarity problem (see [1–10]) is to find a vector $x \in \Re^n$ such that

$$x \geq 0, \quad M(\omega)x + q(\omega) \geq 0, \quad x^T\big[M(\omega)x + q(\omega)\big] = 0, \quad \text{a.e. } \omega \in \Omega, \tag{1.1}$$

where $\Omega \subset \Re^n$ is the underlying sample space and $\omega \in \Omega$ is a random vector with given probability distribution $\mathcal{P}$ and, for each ω, $M(\omega) \in \Re^{n\times n}$ and $q(\omega) \in \Re^n$.

Problem (1.1) is usually denoted by SLCP$(M(\omega), q(\omega))$ or SLCP, briefly. If Ω is a singleton, SLCP reduces to the intensively studied and standard linear complementarity problem (denoted by LCP); see [11–14].

In general there is no vector x satisfying (1.1) for all $\omega \in \Omega$. In order to obtain a reasonable solution of Problem (1.1), there have been several types of models being proposed. One of them is the expected value (EV) model [15] that formulates (1.1) as follows: Let $\bar{M} = E[M(\omega)]$ and $\bar{q} = E[q(\omega)]$ be mathematical expectations of $M(\omega)$ and $q(\omega)$, respectively. The EV model is to find an $x \in \Re^n$ such that

$$x \geq 0, \quad \bar{y} = \bar{M}x + \bar{q} \geq 0, \quad x^T\bar{y} = 0. \tag{1.2}$$

Another is the expected residual minimization (ERM) model (see [1,7]). The ERM model is to find an $x \in \Re^n_+$ that minimizes the expected total residual function

$$\min_{x\geq 0} f(x) = E\big[\|\tilde{\Phi}(x,\omega)\|^2\big] = \sum_{i=1}^{n} E\big\{[\varphi(x_i, M_i(\omega)x + q_i(\omega))]^2\big\}, \tag{1.3}$$

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where $M_i(\omega)\,(i=1,\cdots,n)$ is the i-th row of random matrix $M(\omega)$ and $\varphi:\Re^2\to\Re$ is an NCP-function that satisfies

$$\varphi(a,b)=0 \iff a\geq 0,\ b\geq 0,\ ab=0$$

and

$$\tilde{\Phi}(x,\omega)=\begin{pmatrix}\varphi(x_1,M_1(\omega)x+q_1(\omega))\\ \vdots\\ \varphi(x_n,M_n(\omega)x+q_n(\omega))\end{pmatrix}.$$

Recently, Zhou and Caccetta (see [16]) present a new model for a class of stochastic linear complementarity problems in which sample space Ω has only finitely many elements. Let $\Omega=\{\omega_1,\omega_2,\cdots,\omega_m\}$ and their model is to find an $x\in\Re^n$ such that

$$x\geq 0,\quad M(\omega_i)x+q(\omega_i)\geq 0,\quad x^T\big[M(\omega_i)x+q(\omega_i)\big]=0,\quad i=1,\cdots,m,\ m>1. \tag{1.4}$$

In their model it is assumed that $p_i=\mathcal{P}\{\omega_i\in\Omega\}>0$, $i=1,\cdots,m$, and let $\bar{M}$ and $\bar{q}$ be the expectation values of the random matrix $M(\omega)$ and random vector $q(\omega)$, i.e.,

$$\bar{M}=\sum_{i=1}^m p_iM(\omega_i),\quad \bar{q}=\sum_{i=1}^m p_iq(\omega_i). \tag{1.5}$$

They claim that problem (1.4) is equivalent to (1.6)-(1.7):

$$x\geq 0,\quad \bar{M}x+\bar{q}\geq 0,\quad x^T(\bar{M}x+\bar{q})=0, \tag{1.6}$$
$$M(\omega_i)x+q(\omega_i)\geq 0,\quad i=1,\cdots,m. \tag{1.7}$$

Furthermore, they define

$$\Phi_\alpha(x)=\begin{pmatrix}\varphi_\alpha(x_1,(\bar{M}x+\bar{q})_1)\\ \vdots\\ \varphi_\alpha(x_n,(\bar{M}x+\bar{q})_n)\end{pmatrix},$$

where, $\varphi_\alpha(a,b)=a+b-\sqrt{a^2+b^2}+\alpha[a]_+[b]_+$ with $\alpha>0$ and $[t]_+=\max\{0,t\}$. Then problem (1.4), if it has a solution, can be reformulated as the following minimization problem with nonnegative constraints

$$\begin{aligned}&\min\quad \theta(z)=\frac{1}{2}\|\tilde{H}(z)\|^2,\\ &\text{s.t.}\quad z\geq 0,\end{aligned} \tag{1.8}$$

where $z=(x,y)\in\Re^n\times\Re^{mn}$ and

$$\tilde{H}(z):=\tilde{H}(x,y)=\begin{pmatrix}\Phi_\alpha(x)\\ \tilde{M}(\omega)x+\tilde{q}(\omega)-y\end{pmatrix}.$$

Here

$$\tilde{M}(\omega)=\begin{pmatrix}M(\omega_1)\\ \vdots\\ M(\omega_m)\end{pmatrix}\in\Re^{mn\times n},\quad \tilde{q}(\omega)=\begin{pmatrix}q(\omega_1)\\ \vdots\\ q(\omega_m)\end{pmatrix}\in\Re^{mn}.$$

The authors of [16] propose a semismooth Newton method for solving the constrained minimization problem (1.8). They also examined the effectiveness of the algorithm by means of numerical experiments.