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ESTIMATION OF OPTIMAL ACOUSTIC LINER IMPEDANCE FACTOR FOR REDUCTION OF RADIATED ENGINE NOISE

YANZHAO CAO, M. Y. HUSSAINI, AND HONGTAO YANG

Abstract. We study the optimal design problem of acoustic liner to minimize fan noise radiation from commercial aircraft engine nacelles. Specifically we treat the liner impedance factor as a parameter and seek to estimate its optimal value that minimizes far-field radiated noise. The existence of such an optimal parameter is proved under the assumption that the Helmholtz equation governs the noise field. We also present numerical results to demonstrate that the choice of the optimal liner impedance factor does result in significant reduction of noise level in the far-field.

Key Words. liner impedance factor, the Helmholtz equation, optimization problem

1. Introduction

With dynamic growth in aviation forecast well into the 21st century, aircraft noise will remain a challenging environmental problem. Engine noise being a major component of aircraft noise, interest in inlet and acoustic liner design appears to endure([7, 6, 19]). Minimization of fan noise radiation from commercial aircraft engine nacelles may be achieved by (i) acoustic shape optimization of the inlet and (ii) impedance optimization of the liner. The former problem was studied in [3] using a gradient-based method within the context of a nonprogressive wave environment governed by a Helmholtz equation. The existence of optimal shape is proven, which is obtained numerically by spectral element method, and it yielded 25% noise reduction. As a robust and efficient alternative to gradient-based methods, surrogate management framework method is proposed in [16] for shape optimization of a trailing edge flow to control aerodynamic noise. Liner impedance optimization was studied in [5] using a finite duct noise propagation and radiation code based on boundary integral equation method. It was also investigated within the framework of linearized full potential equation (in the frequency domain) and its discrete adjoint in [21].

In this paper, we treat liner impedance optimization as an optimal control and parameter estimation problem. The parameter is the acoustic impedance factor of the acoustic liner. We define a cost function that reflects the amount of noise radiated from the engine inlet. The parameter estimation problem then is to seek the parameter that minimizes the cost function. The focus of the paper is both

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the mathematical analysis and numerical simulation. We show that an optimal parameter exists mathematically. We also show that the spectrum of the state equation is located in the lower half of the complex plane, which guarantees the stability of our numerical algorithm. To find the optimal parameter numerically, we use the finite element method to seek the numerical solution of state equation and an optimization subroutine to find the minimizer of the cost function. Our numerical result indicates that the choice of optimal parameter results in reducing the noise level by about 40%. The technological feasibility of such a liner material is a different issue that we cannot address.

The paper is organized as follows. In §2, we introduce the optimal control problem for noise reduction. In §3, we establish the solution existence of the optimal control problem. The numerical results are presented in the last section, §4.

2. An optimal control problem

We assume the problem to be axisymmetric. The geometry of the domain in which the control problem is posed has the generic shape represented in Figure 1. The modal composition of the noise source is supposed to be known on the source plane Γ_1. The nacelle boundary is made up of two parts, the first part being the interior boundary Γ_2 to which some acoustic liner material is attached, and the second part being Γ_3 that constitutes the rest of boundary of the nacelle geometry. The boundary Γ_4 is assumed to be sufficiently far from the noise source so that the Sommerfeld radiation boundary condition holds. The nacelle symmetry axis is denoted by Γ_5.

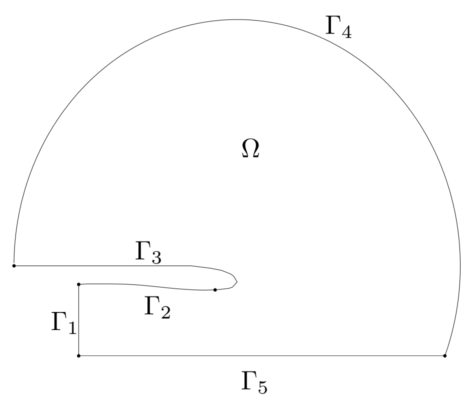

FIGURE 1. The computational domain

If the meanflow is uniform with Mach number M_0, then the governing equation for the acoustic pressure u ([11]) is

$$(1 - M_0^2)\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ikM_0\frac{\partial u}{\partial x} + k^2 u = 0, \tag{2.1}$$

where k is the wavenumber. For simplicity of the presentation, we assume that the mean flow is zero. Then the acoustic pressure u satisfies the Helmholtz equation

$$\Delta u + k^2 u = 0 \quad \text{on } \Omega \tag{2.2}$$