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HIGH RESOLUTION SCB SCHEME FOR HYPERBOLIC SYSTEMS OF 2-D CONSERVATION LAWS*1)

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Abstract

In this paper, a new class of high resolution schemes satisfying the "condition A" (SCA) and the "condition B" (SCB) for hyperbolic systems of conservation laws in one and two dimensions are constructed. Moreover, the results of the numerical experiments by using these schemes are given for the system of Euler equations in one and two dimensions.

Key words: SCB scheme, hyperbolic system, conservation law

1. Introduction

Consider numerical solutions of the initial value problem for hyperbolic conservation laws in one dimension

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{u})}{\partial x} = 0 \tag{1.1a}$$

$$\mathbf{u}(x,0) = \mathbf{u}_0(x) \tag{1.1b}$$

where $\mathbf{u} = (u_1, u_2, \cdots, u_m)^T$ and $\mathbf{f}(\mathbf{u}) = (f_1(\mathbf{u}), f_2(\mathbf{u}), \cdots, f_m(\mathbf{u}))^T$.

And conservation laws in two dimensions

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{u})}{\partial x} + \frac{\partial \mathbf{g}(\mathbf{u})}{\partial y} = 0 \tag{1.2a}$$

$$\mathbf{u}(x,y,0) = \mathbf{u}_0(x,y) \tag{1.2b}$$

where $\mathbf{u} = (u_1, u_2, \cdots, u_m)^T$, $\mathbf{f}(\mathbf{u}) = (f_1(\mathbf{u}), f_2(\mathbf{u}), \cdots, f_m(\mathbf{u}))^T$, and $\mathbf{g}(\mathbf{u}) = (g_1(\mathbf{u}), g_2(\mathbf{u}), \cdots, g_m(\mathbf{u}))^T$.

For the scalar conservation laws in one dimension, the TVD concept by A. Harten[3] is widely accepted to design the numerical schemes for theoretical purposes and practical applications. The total variation of a grid function $\{u_j\}$ denoted by $TV(u)$ is defined as

$$TV(u) = \sum_j \mid u_{j+1} - u_j \mid . \tag{1.3}$$

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Let $BV(R)$ be the space of functions with bounded variation. A difference scheme

$$L(u^{n+1}) = R(u^n),$$

is called as TVD scheme, if for any $u^n \in BV(R)$

$$TV(u^{n+1}) \leq TV(u). \tag{1.4}$$

Encouraged by the success of the TVD schemes in $1-D$, one wants to extend the TVD schemes for two dimensions. The total variation of a grid function $\{u_{j,k}\}$ denoted by $TV(u)$ is defined as

$$TV(u) = \sum_{j,k} [\Delta y \mid u_{j+1,k} - u_{j,k} \mid + \Delta x \mid u_{j,k+1} - u_{j,k} \mid]. \tag{1.5}$$

Same as the case in $1-D$, a difference scheme is called as TVD scheme, if for any $u^n \in BV(R^2)$

$$TV(u^{n+1}) \leq TV(u^n). \tag{1.6}$$

Unfortunately, any conservative TVD scheme for solving scalar conservation laws in two dimensions is at most first order accurate[2]. Hence, it may be worthy of creating new conception beyond TVD in two dimensions. In [7], we have developed a new kind of total variation stability criteria, so-called the "condition A" and the "condition B", and discussed the relationship between them and the TVD conditions. In this paper, we construct a class of high resolution schemes satisfying the "condition A" (SCA) and the "condition B" (SCB) for hyperbolic systems of conservation laws in one dimension and two dimensions. Lastly, some numerical results for the system of Euler equations are given in one and two dimensions.

The organization of this paper is as follows. In section 2, we briefly review the theoretical results in [7]. In section 3 and 4, we construct second order accurate SCA and SCB schemes in one and two dimensions to hyperbolic systems of conservation laws respectively. In section 5, we give the numerical results for the system of Euler equations in one and two dimensions.

2. The "Condition A" and the "Condition B"

In this section, we review the theory developed in [7] for scalar conservation laws ($m = 1$ in (1.1) and (1.2)).

First, let us discuss the conservative schemes in one dimension. Partition the space $[0, T] \times [-\infty, \infty]$. Let Δt and Δx be the temporal and spatial step lengths, respectively. Denote the numerical solution at the point (x_j, t^n) by u_j^n. $x_j = j\Delta x$, $t^n = n\Delta t$ ($j = 0, \pm 1, \pm 2, \cdots$, $n = 0, 1, 2, \cdots$), $\lambda = \Delta t / \Delta x$.

A conservative scheme can be written as following:

$$u_j^{n+1} = u_j^n - \lambda(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}) \tag{2.1}$$