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THE PARTIAL PROJECTION METHOD IN THE FINITE ELEMENT DISCRETIZATION OF THE REISSNER-MINDLIN PLATE MODEL*1)

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Abstract

In the paper a linear combination of both the standard mixed formulation and the displacement one of the Reissner-Mindlin plate theory is used to enhance stability of the former and to remove "locking" of the later. For this new stabilitized formulation, a unified approach to convergence analysis is presented for a wide spectrum of finite element spaces. As long as the rotation space is appropriately enriched, the formulation is convergent for the finite element spaces of sufficiently high order. Optimal-order error estimates with constants independent of the plate thickness are proved for the various lower order methods of this kind.

1. Introduction

It is well known that the standard finite element discretizations of the Reissner-Mindlin plate problem produce poor approximations when the thickness is too small in comparison with the diameter of the region occupied by the midsection of the plate. The root is the so-called "locking" phenomenon which is by now well understood. Among several approaches to avoiding locking is a modification of the standard finite element schemes by interpolating or projecting the discrete transverse shear force into a lower-order finite element space. This kind of method has recently attracted strong research interest due to convenience of implementation and theoretical, experiential evidence.For the details, see [1], [3-5], [9-14] and the papers refered therein.

Both the projection method[1][11][12] and the interpolation method[3−5] are based on the introduction of the shear strain as a new variable. The benefit is that the nonuniform boundedness of the original Reissner-Mindlin variational functional changes into the uniform boundedness of the corresponding Lagrange energy functional with respect to the thickness. As is well known, the nonuniform boundedness of the original formulation leads to locking. But the strong point, i.e. the uniform boundedness of the Lagrange functional is obtained at the expense of the loss of the quadric term of

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primitive shear strain. Therefore it also leads to lack of coerciveness of the resulting discrete formulation which then turns into the difficulty of constructing stable finite element spaces. Based on this analysis, a further modification is considered in the paper. The method here is to divide the discrete shear force into two parts and to project only one of both into a lower-order finite element space. From the point of view of the generalized variational principle, the method can be interpreted as a discretization based on the combination of the original Reissner-Mindlin and the mixed variational principles. To distinguish this from the method in which the whole shear force is projected, the new method is refered to as the partial projection method. For this, a detailed discussion is given in the second section of the paper.

It will be shown that this formulation can be regarded as a reduction of the Hughes-Franca's stabilization technique[11]. More precisely, neglecting one of the two addtional stability terms in the Franca-Hughes's formulation, i.e. removing the least-square residual form of the moment equilibrium equation from their formulation, and expanding the remaining term give essentially the same terms that we get in a equivalent expression of the partial projection formulation. The difference is that the stabilization parameter here appears as a weighted factor independent of the plate thickness and finite element size, but the parameter in [11] is not so. In addition, this reduction leads to a method of Petrov-Galerkin type turning into one of Galerkin type which has foundation of variational principle.

For the case of the deflection interpolations of degree ≥ 2, the partial projection formulation is less versatile than the Hughes-Franca method and the finite element spaces can not be quite arbitrarily chosen, but additional convergence condition can be satisfied much more easily than that required in other mixed methods. As long as the rotation interpolation space is enriched with suitable bubble functions so that a quite simple inf-sup condition holds, this reduced formulation is convergent for any combination of the rotation, deflection and the shear force finite element spaces of sufficiently high order. In particular, the present approach avoids the restrictive condition $\nabla D_h \subset H_h$ required in most mixed methods (where D_h, H_h denote the deflection and the shear force finite element spaces respectively). This condition or its variants lead to rather severe difficulty so that the convergence analysis in the papers [1] [9] [12] can not be extended from the triangular plate elements to equally simple quadrilateral counterparts.

The features mentioned above are confirmed by two general convergence theorems established in section 3 and section 5. In the remaining sections, various applications are discussed. In particular a family of triangular and quadrilateral,conforming and nonconforming Reissner-Mindlin plate elements of order one is constructed, which includes the elements subjected to the discrete Kirchhoff constraint, and for every pair of the triangular and quadrilateral elements, optimal error estimates are achieved in a unified framework with constants independent of the plate thickness.

Following the paper [13], further modification of the methods of order one is con-