SPECTRAL METHOD FOR SOLVING THE RLW EQUATION*

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§ 1. Introduction

The earliest work for the numerical solution of the RLW equation was due to Peregrine[15] and following that Abdulloev, Bogolubsky and Makhankov[1] showed in their numerical experiments the inelastic interaction of soliton-like waves, which is different from that of the KDV equation. Later, Olver[14] proved that the RLW equation possesses only three conserved quantities justifying the inelasticity of the interactions. A lot of numerical work has been done on this equation and the interested reader is referred to Eilbeck and McGuire[5,6] and Alexander and Morris[3]. Recently Wu Hua-mo and Guo Ben-yu[19] have proposed a new high order accurate difference scheme for the KDV-Burgers-RLW equation with a strict error estimation from which the convergence followed. However, when using finite difference schemes or finite element schemes we get only implicit methods and the accuracy of the approximate solution is limited for a fixed scheme, even though the solution of the RLW equation is very smooth.

On the other hand, the above deficiency may be remedied by the use of spectral methods for such problems. In the past several authors (Gazdag[8], Tappert[17], Schamel and Elsässer[16], Canosa and Gazdag[4], Watanabe, Ohishi and Tanaca[18], Fornberg and Whitham[7] and Abe and Inoue[2]) have used spectral methods for such equations and in some recent papers Guo Ben-yu[10,11] has proposed a technique to strictly estimate the error of a spectral method for nonlinear partial differential equations.

This paper is devoted to the use of a spectral method for solving the RLW equation. In Sections 2 and 3, we consider the linear and the nonlinear problems respectively. The corresponding schemes are explicit and the smoother the solution of the RLW equation, the more accurate the approximate solutions are. In Section 4, we report the numerical results obtained for the solutions of the linear and the nonlinear problems and also compare some of these results with those obtained for finite difference and finite element schemes.

§ 2. The Linear RLW Equation

Firstly, we consider the linear RLW equation

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$$\begin{cases} \dfrac{\partial U}{\partial t}+\alpha\dfrac{\partial U}{\partial x}-\delta\dfrac{\partial^3 U}{\partial t\,\partial x^2}=f(x,\,t), & 0<x<2,\ 0<t\leqslant T, \\ U(0,\,t)=U(2,\,t), & 0\leqslant t\leqslant T, \\ U(x,\,0)=U_0(x), & 0<x<2, \end{cases} \tag{1}$$

where α and δ (>0) are constants and $f(0,\,t)=f(2,\,t)$. Let C_i denote a positive constant and

$$A_\alpha=\{U(x,\,t)/|U(x+y,\,t)-U(x,\,t)|\leqslant C_1|y|^\alpha,\quad 0\leqslant t\leqslant T\}.$$

We suppose that (1) has a unique classical solution such that

$$\partial^p U/\partial x^p\in A_\alpha. \tag{2}$$

Put

$$U(x,\,t)=A_0(t)/2+\sum_{l=1}^{\infty}(A_l(t)\cos l\pi x+B_l(t)\sin l\pi x),$$

$$f(x,\,t)=f_0(t)/2+\sum_{l=1}^{\infty}(f_l(t)\cos l\pi x+g_l(t)\sin l\pi x),$$

$$U^{(n)}(x,\,t)=A_0(t)/2+\sum_{l=1}^{n}(A_l(t)\cos l\pi x+B_l(t)\sin l\pi x),$$

and

$$f^{(n)}(x,\,t)=f_0(t)/2+\sum_{l=1}^{n}(f_l(t)\cos l\pi x+g_l(t)\sin l\pi x).$$

Let

$$R^{(n)}(U(x,\,t))=U(x,\,t)-U^{(n)}(x,\,t)$$

and

$$R^{(n)}(f(x,\,t))=f(x,\,t)-f^{(n)}(x,\,t).$$

From Jackson's theorem and Lebésque's theorem (see [20]), we have

$$|R^{(n)}(\partial^r U/\partial x^r)|\leqslant C_2\frac{\ln n}{n^{p+\alpha-r}},\ \text{for}\ \alpha>0. \tag{3}$$

Let τ be the mesh spacing in time and

$$\eta_t(x,\,k\tau)=\frac{1}{\tau}(\eta(x,\,k\tau+\tau)-\eta(x,\,k\tau)),\quad k\geqslant 0.$$

Also let

$$u^{(n)}(x,\,k\tau)=a_0^{(n)}(k\tau)/2+\sum_{l=1}^{n}(a_l^{(n)}(k\tau)\cos l\pi x+b_l^{(n)}(k\tau)\sin l\pi x) \tag{4}$$

be the approximation of $U^{(n)}(x,\,k\tau)$ satisfying

$$\begin{cases} u_t^{(n)}(x,\,k\tau)+\alpha\dfrac{\partial u^{(n)}}{\partial x}(x,\,k\tau)-\delta\dfrac{\partial^2 u_t^{(n)}}{\partial x^2}(x,\,k\tau)=f^{(n)}(x,\,k\tau), & 0<x<2,\ \ k\geqslant 0, \\ u^{(n)}(0,\,k\tau)=u^{(n)}(2,\,k\tau),\quad k\geqslant 0, \\ u^{(n)}(x,\,0)=U_0^{(n)}(x),\quad 0<x<2, \end{cases} \tag{5}$$

i.e.

$$\begin{cases} \dfrac{a_l^{(n)}(k\tau+\tau)-a_l^{(n)}(k\tau)}{\tau}(1+\delta l^2\pi^2)+\alpha l\pi b_l^{(n)}(k\tau)=f_l^{(n)}(k\tau), & 0\leqslant l\leqslant n, \\ \dfrac{b_l^{(n)}(k\tau+\tau)-b_l^{(n)}(k\tau)}{\tau}(1+\delta l^2\pi^2)-\alpha l\pi a_l^{(n)}(k\tau)=g_l^{(n)}(k\tau), & 1\leqslant l\leqslant n, \\ a_l^{(n)}(0)=A_l(0),\quad 0\leqslant l\leqslant n, \\ b_l^{(n)}(0)=B_l(0),\quad 1\leqslant l\leqslant n. \end{cases} \tag{6}$$