# FINITE DIFFERENCE SOLUTIONS OF THE NONLINEAR MUTUAL BOUNDARY PROBLEMS FOR THE SYSTEMS OF FERRO-MAGNETIC CHAIN*

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## §1

The Landau-Lifschitz equation for one-dimensional isotropic Heisenberg ferro-magnetic chain

$$\boldsymbol{s}_t=\boldsymbol{s}\times\boldsymbol{s}_{xx}+\boldsymbol{s}\times\boldsymbol{h} \tag{1}$$

is a strongly degenerate parabolic system, where $\boldsymbol{s}=(s_1, s_2, s_3)$ is a three-dimensional unknown vector function, $\boldsymbol{h}=(0, 0, h(t))$, $h(t)$ is a constant or a function of $t$, "$\times$" denotes the cross-product operator of two three-dimensional vectors[1-4]. In [5] the weak solutions of the periodic boundary problems and the initial problems for more general systems of ferro-magnetic chain

$$z_t=z\times z_{xx}+f(x, t, z) \tag{2}$$

are constructed, where $z=(u, v, w)$ and $f(x, t, z)$ are three-dimensional vector functions. In [6] some simple boundary problems for the system (2) are considered and their finite difference solutions are obtained in [7]. For the systems of ferro-magnetic chain with several variables

$$z_t=z\times\Delta z+f(x, t, z), \tag{3}$$

the homogeneous boundary problem is studied in [8], where $x=(x_1, x_2, \cdots, x_n)$.

In the present work for the system (2) of ferro-magnetic chain the nonlinear mutual boundary problem

$$\begin{aligned} z_x(0, t)&=\mathrm{grad}_0\,\psi(z(0, t), z(l, t)),\\ -z_x(l, t)&=\mathrm{grad}_1\,\psi(z(0, t), z(l, t)) \end{aligned} \tag{4}$$

with the initial condition

$$z(x, 0)=\varphi(x) \tag{5}$$

is considered in the rectangular domain $Q_T=\{0\leqslant x\leqslant l, 0\leqslant t\leqslant T\}$, by means of finite difference method, where $\psi(z_0, z_1)$ is a scalar function of two three-dimensional vector variables $z_0, z_1\in\mathbf{R}^3$, $\varphi(x)$ is a three-dimensional vector function and "$\mathrm{grad}_0$" and "$\mathrm{grad}_1$" denote the gradient operators with respect to $z_0$ and $z_1$ respectively.

Suppose that the following assumptions for the systems (2) of ferro-magnetic chain, the nonlinear mutual boundary conditions (4) and the initial vector function $\varphi(x)$ are valid.

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(I) $f(x, t, z)$ is a three-dimensional continuous vector function for $(x, t, z)\in Q_T\times\mathbb{R}^3$, $f_z(x, t, z)$ is also continuous and $f(x, t, z)$ satisfies the condition of semiboundedness

$$(z-y)\cdot(f(x, t, z)-f(x, t, y))\leqslant b|z-y|^2, \tag{6}$$

where $(x, t)\in Q_T$, $z$, $y\in\mathbb{R}^3$ and $b$ is a constant.

(II) $\psi(z_0, z_1)$ is a continuously differentiable with respect to vector variables $z_0$, $z_1\in\mathbb{R}^3$.

(III) $\varphi(x)\in H^1(0, l)$.

Let us divided the rectangular domain $Q_T$ into small grids by the parallel lines $x=x_j$ $(j=0, 1, \cdots, J)$ and $t=t_n$ $(n=0, 1, \cdots, N)$, where $x_j=jh$, $t_n=n\Delta t$ and $Jh=l$, $N\Delta t=T$. Denote the three-dimensional discrete vector function on the grid point $(x_j, t_n)$ by $z_j^n$ $(j=0, 1, \cdots, J;\ n=0, 1, \cdots, N)$.

Corresponding to the system (2) of ferro-magnetic chain we construct the finite difference system

$$\frac{z_j^n-z_j^{n-1}}{\Delta t}=z_j^n\times\frac{\Delta_+\Delta_-z_j^n}{h^2}+f_j^n,\quad j=1, 2, \cdots, J-1;\ n=1, 2, \cdots, N, \tag{7}$$

where $f_j^n=f(x_j, t_n, z_j^n)$ and $\Delta_+z_j=z_{j+1}-z_j$, $\Delta_-z_j=z_j-z_{j-1}$. The finite difference boundary conditions corresponding to the nonlinear mutual boundary conditions (4) are as follows:

$$\frac{u_1^n-u_0^n}{h}=\frac{\psi(u_1^n, v_1^n, w_1^n; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)-\psi(u_1^{n-1}, v_1^n, w_1^n; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)}{u_1^n-u_1^{n-1}},$$

$$\frac{v_1^n-v_0^n}{h}=\frac{\psi(u_1^{n-1}, v_1^n, w_1^n; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)-\psi(u_1^{n-1}, v_1^{n-1}, w_1^n; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)}{v_1^n-v_1^{n-1}},$$

$$\frac{w_1^n-w_0^n}{h}=\frac{\psi(u_1^{n-1}, v_1^{n-1}, w_1^n; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)-\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)}{w_1^n-w_1^{n-1}},$$

$$-\frac{u_J^n-u_{J-1}^n}{h}=\frac{\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^n, v_{J-1}^n, w_{J-1}^n)-\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^{n-1}, v_{J-1}^n, w_{J-1}^n)}{u_{J-1}^n-u_{J-1}^{n-1}},$$

$$-\frac{v_J^n-v_{J-1}^n}{h}=\frac{\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^{n-1}, v_{J-1}^n, w_{J-1}^n)-\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^{n-1}, v_{J-1}^{n-1}, w_{J-1}^n)}{v_{J-1}^n-v_{J-1}^{n-1}},$$

$$-\frac{w_J^n-w_{J-1}^n}{h}=\frac{\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^{n-1}, v_{J-1}^{n-1}, w_{J-1}^n)-\psi(u_1^{n-1}, v_1^{n-1}, w_1^{n-1}; u_{J-1}^{n-1}, v_{J-1}^{n-1}, w_{J-1}^{n-1})}{w_{J-1}^n-w_{J-1}^{n-1}},$$

$$\tag{8$_1$}$$

where $n=1, 2, \cdots, N$. Denote (8) for brevity by

$$\frac{\Delta_+z_0^n}{h}=\widetilde{\mathrm{grad}}_0\,\psi(z_1^n, z_{J-1}^n),$$
$$-\frac{\Delta_-z_J^n}{h}=\widetilde{\mathrm{grad}}_1\,\psi(z_1^n, z_{J-1}^n). \tag{8$_2$}$$

The finite difference initial condition is

$$z_j^0=\varphi_j,\quad j=0, 1, \cdots, J, \tag{9}$$

where $\varphi_j=\varphi(x_j)$, $j=0, 1, \cdots, J$.

Symbol "$\cdot$" denotes the scalar product of two three-dimensional vectors. For the discrete functions $\{u_j\}$ and $\{v_j\}$, we take the notations:

$$(u\cdot v)_h=\sum_{j=0}^{J}(u_j\cdot v_j)h \quad\text{and}\quad \|u\|_h^2=(u\cdot u)_h.$$