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# Pluriclosed Flow on Oeljeklaus-Toma Manifolds

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**The authors warmly dedicate this article to Professor Gang Tian on the occasion of his 65th birthday.**

**Abstract.** We establish global existence of the pluriclosed flow with arbitrary initial data on Oeljeklaus-Toma manifolds, and Gromov-Hausdorff convergence of blowdown limits to a torus under natural conjectural bounds on the flow at infinity. In the case of generalized Kähler-Ricci flow we prove refined a priori estimates in support of these conjectural bounds.

**AMS subject classifications**: 53E30

**Key words**: Pluriclosed flow, Oeljeklaus-Toma manifolds, Long-time existence, Parabolic estimate.

## 1 Introduction

In recent years the pluriclosed flow [27, 29] and generalized Kähler-Ricci flow [3, 25, 28] have been developed as a tool for understanding the geometry of complex, especially non-Kähler, manifolds [5–7, 10–12, 32]. A natural class of non-Kähler manifolds are the Oeljeklaus-Toma (OT) manifolds [22], whose geometry is linked to the structure of number fields, and which are natural higher dimensional generalizations of Inoue surfaces [18]. In [11] a complete description of the pluriclosed flow with left-invariant initial data on OT manifolds was obtained, in particular showing that the solution exists for all time and collapses after blowdown to a torus in the Gromov-Hausdorff sense. Moreover the blowdown on the universal cover converges in the Cheeger-Gromov sense to a soliton. It is natural to conjecture that these statements hold for arbitrary initial data (Conjecture 3.1). In this work we confirm some aspects of this conjecture.

The first main result is to establish the global existence of the flow:

**Theorem 1.1.** *Fix $(M^{2n}, J)$ an Oeljeklaus-Toma manifold and $g_0$ a pluriclosed metric on $M$. The solution to pluriclosed flow with initial data $g_0$ exists on $[0,\infty)$.*

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The proof exploits natural class of background metrics arising from the homogeneous structure on OT-manifolds. In the case of Inoue surfaces these are known as Tricerri metrics [30]. In the next theorem we give some refined estimates in the special case of generalized Kähler-Ricci flow (GKRF) [28]. Oeljeklaus-Toma manifolds admit natural classes of generalized Kähler structures with split tangent bundle, and for such metrics the GKRF reduces to a scalar parabolic flow of Monge-Ampère type [25].

**Theorem 1.2.** *For generalized Kähler-Ricci flow on an Oeljeklaus-Toma manifold*

1. *The scalar potential $\phi$ satisfies*

$$-C \leq \phi \leq Ce^{-t}(1+t).$$

2. *Assuming there exists a Tricerri-type metric in $[\omega_0]$, we have*

$$-Ce^{-t}(1+t) \leq \phi \leq Ce^{-t}(1+t).$$

3. *On Inoue surfaces of type $S_M$, the estimate of item (2) holds. In addition:*

$$-C \leq \dot{\phi} \leq C.$$

4. *On Inoue surfaces of type $S_M$, we have:*

$$\omega(t) \geq C\omega_h(t),$$

*where $\omega_h(t)$ is the model flow with initial data the Tricerri metric $h$.*

# 2 Geometry of Oeljeklaus-Toma manifolds

## 2.1 Definition

In this section, we recall the family of compact non-Kähler complex manifolds constructed by Oeljeklaus-Toma [22]. First, we need some facts from algebraic number theory. Let $K$ be an algebraic number field, i.e., $K \simeq \mathbb{Q}[x]/(f)$, where $f \in \mathbb{Q}[x]$ is a monic irreducible polynomial of degree $s+2t = [K:\mathbb{Q}]$, where

$$s = \#\ \text{real roots}\ , \quad 2t = \#\ \text{complex roots},$$

which for arbitrary given $s$ and $t$ exists by ([22] Remark 1.1).

Now, consider the embedding, $K \hookrightarrow \mathbb{Q}$, given by the roots of $f$: let $a_1, \cdots, a_s \in \mathbb{R}$ be the real roots of $f$, and let $a_{s+t+1} = \bar{a_{s+1}}, \cdots, a_{s+2t} = \bar{a_{s+t}} \in \mathbb{C}$ be the complex roots of $f$. Define the embedding:

$$\sigma_i : K \to \mathbb{C}, \quad x \mapsto a_i.$$