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Dynamical Analysis in a Discrete Fractional Order Harvested Predator-Prey Model Incorporating Fear Effect and Refuge

Siti Nurul Afiyah[1,2], Fatmawati[3,†] and Windarto[3]

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Abstract This paper examines the dynamics of a discrete fractional-order predator-prey model, incorporating the effects of fear, prey refuge, and population harvesting. The fractional-order framework, which accounts for memory-dependent processes, provides a more accurate depiction of biological interactions than traditional integer-order models. Fear is modeled as a factor that reduces predator-prey encounters, while prey refuge offers a sanctuary, altering population dynamics. The inclusion of harvesting further complicates prey population regulation. We identify the system's equilibrium points and conduct a local stability analysis, focusing on the conditions that lead to stability, instability, and bifurcation. Specifically, we examine the Neimark-Sacker bifurcation, where a stable fixed point evolves into a closed periodic orbit. Numerical simulations corroborate the analytical findings, illustrating the pivotal roles that fear, refuge, and harvesting play in shaping system stability and long-term behavior.

Keywords Dynamical analysis, predator-prey model, harvesting, fear effect, refuge, discrete fractional order

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1. Introduction

Mathematical models in ecology have become important tools for understanding complex dynamics in ecosystems. One of the most studied models is the predator-prey model, which is used to explore interactions between predator and prey populations [1]. One of the well-known predator and prey models is the Lotka-Volterra model introduced by Lotka (1925) and Volterra (1926). One intriguing aspect of the predator-prey model is its ability to produce periodic population patterns, where

[†]the corresponding author.
Email address:noeroel@asia.ac.id(S. N. Afiyah), fatmawati@fst.unair.ac.id(F. Fatmawati),windarto@fst.unair.ac.id(W. Windarto)
[1]Doctoral Program of Mathematics and Natural Science, Faculty of Science and Technology, Universitas Airlangga, Surabaya, Indonesia
[2]Department of Computer Science, Institut Teknologi dan Bisnis Asia, Malang 65113, Indonesia
[3]Department of Mathematics, Faculty of Science and Technology, Universitas Airlangga, Surabaya 60115, Indonesia

the numbers of predators and prey rise and fall in repeated cycles. Wild animals like wolves and gazelles or lions and zebras exhibit this [2].

Predator-prey models have been used widely in ecology to understand species interactions, develop conservation strategies, and predict the impacts of environmental change on animal populations. Furthermore, in the last few decades, many researchers have developed the predator-prey model by modifying the model by adding functional response [3, 4], refuge on prey [5–8] and/or refuge on predators [9], fear effects [10–13], cannibalism [15–17], harvesting [18–20], and various other modifications.

Moreover, from a mathematical perspective, the predator-prey model can be represented as a set of differential equations. The system of integer-order differential equations signifies that the rate of population expansion is influenced by factors beyond the current state. Nevertheless, it is an established truth that the rate of population change is influenced by all preceding conditions, a phenomenon referred to as the memory effect. The phenomenon of memory retention is referred to as fractional order. A system is considered memoryless if its output at any given time t is completely determined by the input at the same time t. A memory system, on the other hand, is used to retain the previous input value in order to calculate the current output value [21].

Fractional order differential equation systems have the ability to provide a real picture of biological phenomena compared to integer order. This fractional differential equation model provides a more accurate description than first-order dynamic systems for complex natural dynamics [22]. The development of fractional order predator-prey models has increased researchers' interest in analyzing fractional order dynamical systems [23–28]. Zhang et al. [29] examined the impact of adding fear effect and prey refuge in a Holling type II predator-prey model. Zhang et al. conducted a study on the influence of fear on the model and discovered that fear not only decreases the population density of the predator but also brings stability to the system. Harvesting is one of the additional factors that influence the dynamics of predator and prey populations. Harvesting of populations is a crucial factor in predator-prey dynamics, as highlighted by Sarkar et al. [30] in their study on the subject. Inspired by this study, we did research on a fractional-order harvesting predator-prey model that incorporates the fear effect and refuge for the prey. The model is as follows:

$$\begin{aligned} {}^{C}D^{\alpha}P(t) &= \frac{r_1 P}{1+fQ} - bP^2 - \frac{\nu(1-\delta)PQ}{1+\eta(1-\delta)P} - z_1(1-\delta)P, \\ {}^{C}D^{\alpha}Q(t) &= -r_2 Q + \frac{c\nu(1-\delta)PQ}{1+\eta(1-\delta)P} - z_2 Q, \end{aligned} \tag{1.1}$$

where condition $P(0) > 0, Q(0) > 0$. Here P and Q are the prey and predator populations, respectively, r_1 is the intrinsic growth of the prey, f is the level of fear, b is the intrinsic growth rate per carrying capacity of prey, c is the efficiency of food conversion from prey to predator, z_1 is the harvesting rate of the prey, r_2 is the predator natural death rate, η is the interaction coefficient, ν is the coefficient of predation, and z_2 is the harvesting rate of the predator.

The predator-prey model with a fractional order in continuous time can be expressed as a nonlinear differential equation. Finding the precise solution to the continuous fractional order predator-prey model is a challenging task. Hence, the utilization of a numerical method is necessary to facilitate the identification of an