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Nonlinear Dynamics and Chaos in Fractional-Order Cardiac Action Potential Duration Mapping Model

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Abstract This study introduces a novel one-dimensional fractional-order model for cardiac action potential duration (APD) dynamics, incorporating memory effects through discrete fractional calculus. By generalizing the classical APD map using the Caputo fractional difference operator, we uncover complex nonlinear behaviors not observed in traditional integer-order models. Through comprehensive numerical simulations, including bifurcation analysis and Lyapunov exponent calculations validated by the 0-1 test, we demonstrate that the fractional-order system exhibits:

1) Early onset of chaos (at $t_s = 307ms$) without preceding period-doubling bifurcations.
2) Novel rhythm alternations between $5:5$ and $3:3$ patterns.
3) Unique bistability phenomena, including $2:2 \longleftrightarrow chaos$ and $5:5 \longleftrightarrow 3:3$ states.
4) Memory-dependent dynamics where current APD depends on all previous states.

Our results reveal that fractional calculus provides a more physiologically realistic framework for modeling cardiac dynamics by naturally incorporating memory effects. The identified dynamical regimes offer new insights into the transition mechanisms from normal rhythms to potentially arrhythmic states, with particular clinical relevance to understanding alternans as precursors to ventricular fibrillation. The fractional-order approach demonstrates superior capability for capturing the complex, history-dependent nature of cardiac excitation compared with conventional models.

Keywords Fractional-order calculus, cardiac dynamics, action potential duration, nonlinear dynamics, chaos

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1. Introduction

To understand cardiac arrhythmias in the human heart, it is essential to grasp the complex dynamics represented in mathematical models. The idea that mathematical analysis can aid in understanding these arrhythmias is not a recent development. As early as the 1920s, researchers showed that by adjusting parameters in mathematical models of the heart, they could replicate rhythms that are clinically observed in arrhythmias [12, 24].

The behavior of various cardiac arrhythmias and the transitions between different heart rhythms suggest that nonlinear phenomena play a significant role in the development of these irregular heartbeats. Unlike most cardiac diseases, which progress slowly and steadily over many years, arrhythmias can often occur suddenly. In nonlinear systems, sudden changes in dynamics can unexpectedly arise from gradual modifications to a system parameter; this phenomenon is known as bifurcation [20].

Memory has been the subject of extensive research across multiple disciplines, including physics, chemistry, biology and electrical engineering [1, 4, 17]. In systems characterized by memory, dynamic behavior is significantly influenced by prior experiences, as exemplified by hysteresis observed in ferromagnetic materials. In electrically excitable cells, such as neurons and cardiomyocytes, the dynamics of excitation are governed by intricate networks that encompass a variety of ion channels and signaling pathways, each functioning on distinct time scales. Consequently, these systems frequently exhibit memory effects. For instance, electrical bursting in neurons and pancreatic β-cells arises from interactions between rapid and slow time scales, with the slower time scales potentially contributing to the development of memory.

Low-dimensional iterated maps have been extensively employed to elucidate the dynamic mechanisms underlying complex cardiac excitations. One of the most widely recognized iterated map models is grounded in cardiac myocytes' action potential (AP) duration-restitution characteristics [26]. The concept of action potential duration (APD) restitution is well-established in cardiology and has undergone thorough investigation in various experimental studies [14, 27, 29]. These iterated maps demonstrate effectiveness when the influence of memory is negligible. However, their limitations become apparent when substantial memory effects indicate the need for higher-dimensional maps to capture these dynamics accurately. Previous research has examined the relationship between memory and cardiac alternans, generally concluding that memory exerts a suppressive effect on alternans [15,33,34].

Incorporating the memory effect into a model can be effectively achieved through the use of a fractional-order operator. Over the past four decades, fractional calculus—an extension of continuous-time and discrete-time chaotic dynamical systems to non-integer orders—has garnered significant attention [6–8, 18, 35]. This fascination is largely because fractional-order models provide a more precise representation of complex phenomena and reveal behaviors that integer-order models often overlook. Moreover, fractional calculus naturally aligns with systems characterized by memory, which are frequently found in numerous biological processes [2, 11, 19, 28, 30–32]. Embracing these models could lead to breakthroughs in understanding and analyzing intricate systems.

In this work, we will summarize the contributions made by Lewis and Guevara [23]. To incorporate the memory effect into their proposed model, we introduce a