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A Predator-Prey Biological Model of Multiple Species with Linear Growth Rates

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Abstract The purpose of this paper is to give sufficient conditions for the existence and uniqueness of positive solutions to an elliptic system of the Dirichlet problem on a bounded domain Ω in R^n. Also considered are the effects of perturbations on the coexistence state and uniqueness. The techniques used in this paper are super-sub solutions method, eigenvalues of operators, maximum principles, spectrum estimates, inverse function theory, and general elliptic theory. The arguments also rely on some detailed properties for the solution of logistic equations. These results yield an algebraically computable criterion for the positive coexistence of species of animals with predator-prey relation in many biological models.

Keywords Predator-prey system, coexistence state, existence, uniqueness, perturbation

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1. Introduction

One of the prominent subjects of study and analysis in mathematical biology concerns the survival of two or more species of animals in the same environment. Especially, pertinent areas of investigation include the conditions under which the species can coexist, as well as the conditions under which any one of the species becomes extinct, that is, one of the species is excluded by the others. In this paper, we focus on the predator-prey model to better understand the competitive interactions between multiple species. Specifically, we investigate the conditions needed for the coexistence of species when the factors affecting them are fixed or perturbed.

2. Literature review

Within the academia of mathematical biology, extensive academic work has been devoted to investigation of the simple predator-prey model, commonly known as the Lotka-Volterra predator-prey model. This system describes the predator-prey interaction of two species residing in the same environment in the following manner:

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Suppose two species of animals, one is prey and the other is predator, are residing in a bounded domain Ω. Let $u(x,t)$ be the density of prey and $v(x,t)$ be density of the predator in the place x of Ω at time t. Then we have the following biological interpretation of terms.

(A) The partial derivatives $u_t(x,t)$ and $v_t(x,t)$ mean the rate of change of densities with respect to time t.
(B) The laplacians $\Delta u(x,t)$ and $\Delta v(x,t)$ stand for the diffusion or migration rates.
(C) The rates of self-reproduction of each species of animals are expressed as multiples of some positive constants α, β and current densities $u(x,t), v(x,t)$, i.e. $\alpha u(x,t)$ and $\beta v(x,t)$ which will increase the rate of change of densities in (A), where $\alpha > 0, \beta > 0$ are called the self-reproduction constants.
(D) The rates of self-limitation of each species of animals are multiples of some positive constants a, d and the frequency of encounters among themselves $u^2(x,t)$, $v^2(x,t)$, i.e. $bu^2(x,t)$ and $fv^2(x,t)$ which will decrease the rate of change of densities in (A), where $a > 0, d > 0$ are called the self-limitation constants.
(E) The rates of competition of each species of animals are multiples of some positive constants b, c and the frequency of encounters of each species with the other $u(x,t)v(x,t)$, i.e. $bu(x,t)v(x,t)$ and $cu(x,t)v(x,t)$ which will decrease the rate of change of densities of prey and increase the rate of change of densities of predator in (A), where $b > 0, c > 0$ are called the competition constants.
(F) We assume that none of the species of animals is staying on the boundary of Ω.

Combining all those together, we have the dynamic predator-prey model

$$\begin{cases} u_t(x,t) = \Delta u(x,t) + \alpha u(x,t) - au^2(x,t) - bu(x,t)v(x,t) & \text{in } \ \Omega \times [0,\infty), \\ v_t(x,t) = \Delta v(x,t) + \beta v(x,t) - dv^2(x,t) + cu(x,t)v(x,t) & \\ u(x,t) = v(x,t) = 0 \ \text{ for } \ x \in \partial\Omega, & \end{cases}$$

or equivalently,

$$\begin{cases} u_t(x,t) = \Delta u(x,t) + u(x,t)(\alpha - au(x,t) - bv(x,t)) & \text{in } \ \Omega \times [0,\infty), \\ v_t(x,t) = \Delta v(x,t) + v(x,t)(\beta - dv(x,t) + cu(x,t)) & \\ u(x,t) = v(x,t) = 0 \ \text{ for } \ x \in \partial\Omega. & \end{cases}$$

Here we are interested in the time independent, positive solutions, i.e. the positive solutions $u(x), v(x)$ of

$$\begin{cases} \Delta u(x) + u(x)(\alpha - au(x) - bv(x)) = 0 & \text{in } \ \Omega, \\ \Delta v(x) + v(x)(\beta - dv(x) + cu(x)) = 0 & \\ u|_{\partial\Omega} = v|_{\partial\Omega} = 0, & \end{cases} \tag{2.1}$$

which are called the coexistence state or the steady state. The coexistence state is the positive density solution depending only on the spatial variable x, not on the time variable t, and so its existence means the two species of animals can live peacefully and forever.