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Solving the Kuramoto-Sivashinsky Equation via a Modified Adomian Decomposition Method

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Abstract The Kuramoto-Sivashinsky equation (KSE) is a well-known nonlinear partial differential equation (PDE) that plays a significant role in various scientific fields, particularly in fluid dynamics and reaction-diffusion systems. In this study, we employ the Modified Adomian Decomposition Method (MADM) to derive an analytical solution to the KSE over the domain $t \in \mathbb{R}$ and $0 \leq x < 1$. The research provides a comprehensive analysis of the applicability of MADM in generating explicit series solutions for the KSE. The equation is formulated under proper initial conditions, and the systematic implementation of MADM is demonstrated involving the decomposition of nonlinear terms and successive approximation. The convergence and stability of the resulting series solutions are investigated, highlighting the method's efficiency in capturing the dynamics described by the KSE. Numerical simulations are presented to validate the analytical results, illustrating the effectiveness of the MADM in solving the KSE within the given parameters. This study not only enhances the theoretical understanding of the KSE but also serves as a practical guide for applying analytical techniques to complex nonlinear PDEs. Moreover, the proposed method represents a significant advancement in solving highly non-linear models by providing a straightforward recursive scheme that avoids linearization or small-parameter assumptions. The physical relevance of the obtained solutions is reflected in their ability to capture key features of instability and chaotic behavior characteristic of the KSE. These findings underscore the potential of MADM as a powerful and versatile tool in the study of spatiotemporal phenomena governed by nonlinear PDEs.

Keywords Non-linear evolution equations, decomposition techniques, series solutions, initial value problems

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1. Introduction

The KSE is a prominent fourth-order nonlinear partial differential equation that arises in various fields, including fluid dynamics, combustion theory, and pattern formation. Its complexity and its role in capturing chaotic behavior make it a critical subject of study in applied mathematics and physics. This equation effectively models processes characterized by instability and turbulence. For example, Hyman and

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Nicolaenko [12] demonstrated how the KSE bridges partial differential equations and dynamical systems to capture chaotic and complex spatiotemporal behavior. Cousin and Larkin [5] analyzed the KSE in domains with moving boundaries, highlighting its role in dynamic boundary interactions. Ebiwareme [8] empolyed the tanh-coth method to obtain exact and solitary solutions, illustrating wave propagation and nonlinear wave behavior. Khater and Temsah [14] applied Chebyshev spectral collocation methods to compute accurate numerical solutions of the generalized KSE. Lu et al. [23] proposed data-driven stochastic model reduction approaches for the KSE, supporting its utility in uncertainty quantification and simplified modeling. Shah et al. [27] introduced semi-analytical methods to solve families of KSEs, offering insights into efficient computational handling. Troy [29] addressed the existence of steady-state solutions to the KSE, which is crucial for understanding long-term system behavior. Zavalani [31] implemented exponential time differencing methods to enhance temporal integration accuracy, while Zhonghua et al. [32] developed fully discrete Galerkin methods and provided rigorous error estimates contributing to numerical stability and simulation precision. Recently, various analytical methods have been effectively utilized in combination with decomposition techniques to address a wide range of linear and nonlinear initial and boundary value problems. For instance, recent studies have employed efficient analytical frameworks such as the Aboodh transformation and Laplace based decomposition methods to tackle integro-differential systems and fractional dynamical equations without the need for discretization or perturbation [15, 16]. Moreover, researchers have introduced significant modifications to classical decomposition techniques to enhance their flexibility and accuracy when applied to nonlinear systems [17–19]. These developments include iterative procedures and polynomial-based schemes that improve convergence and reduce computational cost [20, 21]. By incorporating such advancements, this study aligns with recent analytical trends and contributes to the growing body of literature focused on the enhancement of decomposition-based methods for complex differential systems.

Here, we implement the MADM to address the KSE over the domain $t \in \mathbb{R}$ and $0 \leq x < 1$.

We investigate the dynamic behavior of the KSE in a bounded spatial setting, emphasizing the mathematical challenges associated with its nonlinear and higher-order derivatives. The spatial domain is chosen to explore localized phenomena and boundary effects that are significant in many physical applications. Several analytical methods have been developed to study this equation, each addressing different aspects of its solutions and properties. These include: Homotopy Analysis Method (HAM) [6], $\frac{1}{G'}$ expansion method [30] and Adomian Decomposition Method [24].

The Adomian Decomposition Method (ADM) [1] is a powerful and versatile method used for solving nonlinear partial differential equations (PDEs). This technique breaks down the problem into a series of decomposed equations that are easier to solve, making it particularly effective for complex nonlinear systems. It represents the solution as an infinite series, computed recursively through the following key steps [2]:

1.**Function decomposition**: The nonlinear differential equation is expressed in terms of a linear operator and a nonlinear operator. The solution $u(x,t)$ is decomposed into a series, typically as follows:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \dots,$$