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Global Attractors for the BBM Equation with Fading Memory

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Abstract This paper considers the existence of global attractors for BBM equation with fading memory. Using some new estimate technique to prove the existence of global attractors in topological space $H_1 \times L^2_\mu(\mathbb{R}^+; H_1)$.

Keywords BBM equations, global attractors, fading memory

MSC(2010) 35B40, 35B41.

1. Introduction

In this paper, we study the following BBM equation with fading memory:

$$\begin{cases} u_t - \Delta u_t - \alpha\Delta u - \int_0^\infty k'(s)\Delta u(t-s)ds + (g(u))_x = f, & \text{in } \Omega, t > 0, \\ u(t) = u_0(t), & \text{in } \Omega, t \leqslant 0, \\ u_{|\partial\Omega} = 0, & t \in \mathbb{R}, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}$ is a bounded interval of the real line, f is a deterministic time-dependent external forcing term, and memory kernel function $k'(s)$ is assumed to satisfy the posterior adaptation hypothesis, which will be detailed in the subsequent sections. It describes the characteristic that the current state of a material or system is influenced by its past states. $g \in L^2(\Omega)$ is a nonlinear term, and satisfies:

$$g(u) = u + \frac{1}{2}u^2.$$

BBM equation was first proposed by Benjamin, Bona and Mahony, which describes the mathematical model of long waves propagation with nonlinear dispersion and dissipation effects, see [3]. In recent years, the asymptotic behavior of BBM equations has been studied by many authors. In [1, 2, 4, 5, 7], the authors studied the global well-posedness and ill-posedness of BBM equation in H^S and L^P type Sobolev spaces. In [8], the authors proved the existence of global attractors of BBM equations on bounded domains in H^1. Stanislavova et al. proved the existence of global attractors in H^1 for the BBM equation on unbounded domains, see [17, 18]. In [21], Yang et al. proved the upper semi-continuity of the pullback attractor of the three-dimensional non-autonomous BBM equation. Dell 'Oro et al. [11] proved the existence of global attractors of BBM equations with memory. In [19], the authors

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studied the long-term behavior of the generalized BBM equation with damping in a low regularity space.

In recent years, the research on global attractors of equations with fading memory has attracted much attention. For example, in [6, 15] the authors proved the existence of differential equations attractor with fading memory. Zhang et al. [22] showed the existence of strong global attractor for nonclassical diffusion equation with fading memory. Xuan and Munteanu [14,20] respectively proved the attractors of abstract evolution equations and Navier-Stokes Equation with fading memory. The dynamic behavior of the solution of (1.1) in space$H_1 \times L^2_\mu(\mathbb{R}^+; H_1)$ has not yet been considered, and it can be said that it is studied for the first time in this paper as a new problem. In this paper, we will use some new estimate technique to prove the existence of global attractors in topological space $H_1 \times L^2_\mu(\mathbb{R}^+; H_1)$.

The organizational structure of this paper is as follows. In Section 2, we present some notations and proposition of function space. In Section 3, we show the existence of global attractors in $H_1 \times L^2_\mu(\mathbb{R}^+; H_1)$.

2. Preliminaries and abstract results

In this section, we present some notations and proposition of function space.

Set $A = -\Delta$, $H_0 = L^2(\Omega)$, $H_1 = H^1_0(\Omega)$, $H_2 = L^2(\Omega) \cap H^1_0(\Omega)$. Denote as $H_r = D(A^{r/2})$ $(0 \le r \le 2)$, the inner product and norm are as follows:

$$\langle u, v\rangle_{H_r} = \langle A^{r/2}u, A^{r/2}v\rangle, \quad \|u\|_{H_r} = \|A^{r/2}u\|. \tag{2.1}$$

Clearly,

$$H_2 \subset H_1 \subset H_0 = H_0^* \subset H_1^*, \tag{2.2}$$

where H_0^* and H_1^* represent the dual space of H_0, H_1 respectively.

If $\forall m < s$, we have

$$D(A^{\frac{s}{2}}) \subset D(A^{\frac{m}{2}}), \quad D(A^{\frac{s}{2}}) \subset L^{\frac{2n}{n-2s}}(\Omega).$$

Using the Poincaré inequality, we can obtain:

$$\sqrt{\lambda_1}\|v\|_s \le \|v\|_{s+1}, \quad \forall v \in H^1_0. \tag{2.3}$$

Let $\mu(s) = -k'(s)$ and $k(\infty) = 0$. Then by (1.1), we have

$$\begin{cases} u_t - \Delta u_t - \alpha\Delta u - \int_0^\infty \mu(s)\Delta\eta(t-s)ds + (g(u))_x = f, \\ \eta^t_t = -\eta^t_s + u, \end{cases} \tag{2.4}$$

with the initial boundary conditions

$$\begin{cases} u(x,t)|_{\partial\Omega} = 0, \quad \eta^t(x,s)|_{\partial\Omega\times\mathbb{R}^+} = 0, \quad t \ge 0, \\ u(x,0) = u_0(x), \quad \eta^0(x,s) = \int_0^s u_0(x,-\tau)d\tau, \quad (x,s) \in \Omega\times\mathbb{R}^+. \end{cases} \tag{2.5}$$

In (1.1), the role of fading memory is reflected in the function $\Delta u(\cdot)$ and the linear convolution term of the memory kernel function $k(\cdot)$. From [14, 20] we make the following assumptions on memory kernel function μ:

$$\mu \in \mathrm{C}^1(\mathrm{R}^+) \cap \mathrm{L}^1(\mathrm{R}^+), \ \mu'(\mathrm{s}) \le 0 \le \mu(\mathrm{s}), \ \forall \mathrm{s} \in \mathbb{R}^+; \tag{2.6}$$