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ASYMPTOTIC BEHAVIOR OF GLOBAL CLASSICAL SOLUTIONS TO A KIND OF MIXED INITIAL-BOUNDARY VALUE PROBLEM

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Abstract We study the asymptotic behavior of global classical solutions to a kind of mixed initial-boundary value problem for quasilinear hyperbolic systems. Based on the existence results on the global classical solutions given by Li and Wang in [1] and employing the method of Kong and Yang in [2], we prove that, when t tends to infinity, the solution approaches a combination of C^1 travelling wave solutions at the algebraic rate $(1+t)^{-\mu}$, provided that the initial data decay at the rate $(1+x)^{-(1+\mu)}$ as x tends to $+\infty$ and the boundary data decay at the rate $(1+t)^{-(1+\mu)}$ as t tends to $+\infty$, where μ is a positive constant.
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1. Introduction and Main Result

Consider the following first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = 0, \tag{1.1}$$

where $u = (u_1, \cdots, u_n)^T$ is the unknown vector function of (t, x) and $A(u)$ is an $n \times n$ matrix with suitably smooth elements $a_{ij}(u)(i, j = 1, \cdots, n)$.

By the definition of hyperbolicity , for any given u on the domain under consideration, $A_{ij}(u)$ has n real eigenvalues, $\lambda_1(u), \cdots, \lambda_n(u)$ and a complete set of left (resp. right) eigenvectors. For $i = 1, \cdots, n$, let $l_i(u) = (l_{i1}(u), \cdots, l_{in}(u))$ $(resp. r_i(u) = (r_{i1}(u), \cdots, r_{in}(u))^T)$ be a left ($resp.$ right) eigenvector corresponding to $\lambda_i(u)$:

$$l_i(u)A(u) = \lambda_i(u)l_i(u), \tag{1.2}$$

and

$$A(u)r_i(u) = \lambda_i(u)r_i(u). \tag{1.3}$$

We have

$$\det|l_{ij}(u)| \neq 0 \quad (resp.\ \det|r_{ij}(u)| \neq 0). \tag{1.4}$$

Without loss of generality, we suppose that on the domain under consideration

$$l_i(u)r_j(u) = \delta_{ij} \qquad (i, j = 1, \cdots, n), \tag{1.5}$$

where δ_{ij} stands for the Kronecker's symbol .

We suppose that all $\lambda_i(u), l_{ij}(u), r_{ij}(u)$ $(i, j = 1, \cdots, n)$ have the same regularity as $a_{ij}(u)(i, j = 1, \cdots, n)$.

In this paper, we suppose that the eigenvalues satisfy

$$\lambda_1(0), \cdots, \lambda_m(0) < 0 < \lambda_{m+1}(0) < \cdots < \lambda_n(0). \tag{1.6}$$

On the domain

$$D = \{(t, x) \mid t \geq 0, x \geq 0\}, \tag{1.7}$$

we consider the mixed initial-boundary value problem for the system (1.1) with the initial condition

$$t = 0 : \quad u = \varphi(x) \qquad (x \geq 0), \tag{1.8}$$

and the boundary condition

$$x = 0 \ : \quad v_s = f_s(\alpha(t), v_1, \cdots, v_m) + h_s(t) \quad (s = m+1, \cdots, n), \tag{1.9}$$

in which

$$v_i(u) = l_i(u)u \quad (i, = 1, \cdots, n), \tag{1.10}$$

and

$$\alpha(t) = (\alpha_1(t), \cdots, \alpha_k(t)). \tag{1.11}$$

Without loss of generality, we suppose that

$$f_s(\alpha(t), 0, \cdots, 0) \equiv 0 \qquad (s = m+1, \cdots, n). \tag{1.12}$$

Remark 1.1 : In a neighborhood of $u = 0$, the boundary condition (1.9) takes the same form under any possibly different choice of the left eigenvectors.(see [1])

For the Cauchy problem, the following result was proved by Kong and Yang in [2] :

Theorem A *Under the assumptions of above, there exists a unique C^1 vector-valued function $\Phi(x) = (\Phi_1(x), \cdots, \Phi_n(x))^T$ such that in the normalized coordinates (see Section 2.1)*

$$\left| u(t, x) - \sum_{i=1}^{n} \Phi_i(x - \lambda_i(0)t)e_i \right| \ \leq \ K\theta^2(1+t)^{-\mu}, \tag{1.13}$$

where K stands for a positive constant independent of (t, x) and θ, and $\lambda_1(0) < \lambda_2(0) < \cdots < \lambda_n(0)$.