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Lie Symmetries, Conservation Laws, Optimal System and Similarity Reductions of (2+1)-Dimensional Fractional Hirota-Maccari System

YU Jicheng[1,*] and FENG Yuqiang[2]

[1] *School of Science, Wuhan University of Science and Technology, Wuhan 430081, China;*
[2] *Hubei Province Key Laboratory of Systems Science in Metallurgical Process, Wuhan 430081, China.*

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Abstract. In this paper, Lie symmetry analysis method is applied to one type of mathematical physics equations named the (2+1)-dimensional fractional Hirota-Maccari system. All Lie symmetries and the corresponding conserved vectors for the system are obtained. The one-dimensional optimal system is utilized to reduce the aimed equations with Riemann-Liouville fractional derivative to the (1+1)-dimensional fractional partial differential equations with Erdélyi-Kober fractional derivative.

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1 Introduction

Nonelinear partial differential equations are increasingly used to model nonlinear physical phenomena. Among them, the following (2+1)-dimensional Hirota-Maccari system is considered:

$$i\Phi_t+\Phi_{xy}+i\Phi_{xxx}+\Phi\Psi-i|\Phi|^2\Phi_x=0, \qquad 3\Psi_x+(|\Phi|^2)_y=0, \tag{1.1}$$

which was firstly studied by Hirota [1] and Maccari [2]. This system has many physical applications such as femtosecond pulse propagation in optical fibers, the propagation of

*Corresponding author. *Email addresses:* `yjicheng@126.com` (J. C. Yu), `yqfeng6@126.com` (Y. Q. Feng)

optical pulse in nematic liquid crystal waveguides, also shows interaction of lower hybrid large-amplitude waves with finite frequency density perturbations [3]. "It has been demonstrated that the Hirota-Maccari system is stable under small perturbations, as well as integrable due to the presence of Lax pair and because it passes the Painleve test for integrability" [4]. Recently, the classical Hirota-Maccari system (1.1) is extended to different fractional versions, which have the practical physical background and been studied by different methods. In [5], the modified Kudryashov's and the Auxiliary equation methods are used to build a variety of soliton solutions of the Hirota-Maccari system with time fractional derivative, and the simulated graphics are used to explain the real phenomena described by the system arising in diverse fields of science. The authors in [6,7] respectively used the Jacobi elliptic functions approach and dynamic system method to discuss the effect of fractional derivative and multiplicative noise on the dynamic behavior of the stochastic fractional Hirota-Maccari system, which is applied to hydrodynamics, plasma and optical fiber propagation.

In this paper, Eqs. (1.1) are extended to the following time-fractional version:

$$iD_t^{\alpha}\Phi+\Phi_{xy}+i\Phi_{xxx}+\Phi\Psi-i|\Phi|^2\Phi_x=0, \quad 3\Psi_x+(|\Phi|^2)_y=0, \qquad 0<\alpha<1, \tag{1.2}$$

where $\Phi(t,x,y)$ and $\Psi(t,x,y)$ are complex-valued and real-valued functions, respectively, t is the temporal variable and x, y are independent spatial variables. The non locality of time fractional derivative operator is very suitable for describing time delay or memory and genetic effects. Therefore, time fractional Hirota-Maccari system can more accurately describe the dissemination of waves with higher-order dispersion in plasma physics and the characteristics of optical fiber propagation (see [5-7]). Assuming $\Phi=U(t,x,y)+iV(t,x,y)$ and $\Psi=W(t,x,y)$, then Eqs. (1.2) can be rewritten as

$$\begin{cases} -D_t^{\alpha}V_t+U_{xy}-V_{xxx}+UW+V_x(U^2+V^2)=0, \\ D_t^{\alpha}U_t+V_{xy}+U_{xxx}+VW-U_x(U^2+V^2)=0, \\ 3W_x+2UU_y+2VV_y=0. \end{cases} \tag{1.3}$$

As a generalization of the classical calculus, fractional calculus can be traced back to the letter written by L'Hôspital to Leibniz in 1695. Since then, it has gradually gained the attention of mathematicians. Especially in recent decades, it has developed rapidly and been successfully applied in many fields of science and technology [8–11]. Therefore, it is very important to find the solution of fractional differential equation. So far, there have been some numerical and analytical methods, such as Adomian decomposition method [12], finite difference method [13], homotopy perturbation method [14], the sub-equation method [15], the variational iteration method [16], Lie symmetry analysis method [17], invariant subspace method [18] and so on. Among them, Lie symmetry analysis method has received an increasing attention.

Lie symmetry analysis method was founded by Norwegian mathematician Sophus Lie at the end of the nineteenth century and then further developed by some other mathematicians, such as Ovsiannikov [19], Olver [20], Ibragimov [21–23] and so on. As a